Peer effects via socially influenced attention*

Abhinash Borah† Raghvi Garg‡ Ojasvi Khare§ Siddarth Venkatesh¶

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Abstract

We introduce an equilibrium-based model of peer effects in a cluster network of boundedly rational agents. The mechanism underlying peer effects draws on the idea of socially influenced random attention, specifically, the more popular is an alternative amongst an individual's peers, the more likely it is to receive their attention and, consequently, greater their probability of choosing it. We introduce the concept of an *attention through peers* equilibrium to model this choice-attention interaction among peers. We show that such an equilibrium uniquely exists and is supported as the outcome of observational learning. The model permits an exact identification of the cluster network, along with individual preferences and their susceptibilities to influence, which together underlie peer effects. We characterize the model behaviorally, enabling a transparent check for whether empirically observed evidence of peer effects is consistent with our mechanism or not. We show the existence of a social multiplier and highlight a novel preference-attention interaction that drives nonlinearities in the measurement of peer effects and implies a crowding out effect, whereby more preferred alternatives crowd out peer effects of less preferred ones.
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†Ashoka University (abhinash.borah@ashoka.edu.in)
‡Indian Institute of Technology, Kanpur (raghvi@iitk.ac.in)
§Shiv Nadar Institute of Eminence (ojasvi.khare@snu.edu.in)
¶University of Chicago (siddarth@uchicago.edu)
Correspondence address: Department of Economics, Ashoka University. Rajiv Gandhi Education City, Sonepat. Haryana - 131029, India.

1 Introduction

Analyzing the influence of one's peers on behavior has been an enduring concern of economic analysis. When such peer effects exist, a key question of interest is about the mechanism through which such effects work. It has been noted in the literature that identifying mechanisms that drive peer effects is among the more challenging open questions in this line of research and, further, that the analysis of new theoretical models of peer interactions and their subsequent estimation may hold the key to addressing this question (Boucher and Fortin, 2016; Bramoullé, Djebbari, and Fortin, 2020). Motivated by this perspective, this paper studies a natural mechanism that generates peer effects. Our mechanism draws on the notion of *socially influenced random attention*, specifically, the idea that a decision maker's attention or consideration is more likely to be drawn towards alternatives that are popular among her peers, in turn making these alternatives more likely to be chosen.[1] Hence, peer effects according to this perspective is generated by the mutual interactions between peers' attention and choices. We study these peer effects in the context of a cluster network, i.e., a social network in which the set of individuals are partitioned into clusters, and all individuals within a cluster are connected.[2]

We explore this mechanism with several goals in mind. First, we establish the theoretical coherence of the mechanism by (a) proposing an equilibrium notion that pins down what mutual consistency of these peer interactions imply, and (b) establishing that this equilibrium notion is non-vacuous and always exists uniquely. This exercise mirrors for our boundedly rational environment the analysis that has been done in the literature to provide micro-foundation for popular peer effects models like the linear-in-means (LIM) model by modeling it as the (Bayes) Nash equilibrium outcome of a network game among

[1] The observation that, in any given choice problem, a boundedly rational decision maker's attention may be limited to only a subset of the available alternatives is a well-documented behavioral phenomenon in the literature, e.g., Masatlioglu, Nakajima, and Ozbay (2012), Eliaz and Spiegler (2011), Manzini and Mariotti (2014), Brady and Rehbeck (2016), Lleras et al. (2017), Caplin, Dean, and Leahy (2018), and Dardanoni et al. (2020), among others. More recently, a strand of the literature has explored the possibility that social influence may play a role in determining a decision maker's attention in the sense that she may only consider alternatives that her peers choose or they recommend, e.g., Borah and Kops (2018) and Kashaev, Lazzati, and Xiao (2023).

[2] Cluster networks have been the predominant social structure within which peer effects have been studied. Apart from the fact that such clusterings naturally form in society, one reason for this focus is that unlike diffusion where indirect connections formed between neighbors of neighbors may be adequate for propagation, the effects of influence are most prominent in direct connections: "Having a close friend engage in some behavior is likely to have more of an effect on someone than if a friend of a friend engages in that same behavior" (McAdam, 1986). Empirical evidence of peer effects in the context of a cluster network has been reported in several areas, e.g., voting behavior (Cohen, 2003; Barber and Pope, 2019; Macy et al., 2019; Miller and Conover, 2015), formative behavior like smoking, prosocial attitudes, aggressive behavior (Ehlert et al., 2020; Ellis and Zarbatany, 2017; Lodder et al., 2016), body image concerns and dieting behavior (Paxton et al., 1999), and academic performance (Zimmerman, 2003). From an econometric perspective, cluster networks are noteworthy as they raise challenges of identification (Manski, 1993).

rational players; or discrete choice models of influence within a random utility framework.[3] Second, we draw on decision-theoretic tools to address the problem of identification, which is a key challenge in this line of work. Specifically, we exploit the stochasticity in behavior produced by random attention to uniquely identify the cluster network underlying influence, along with individual characteristics like preferences and idiosyncratic susceptibility to influence that underlie peer effects. Third, we behaviorally characterize the model and provide grounds for its falsification, thus enabling a transparent check for whether choice data suggestive of peer influence conforms with our mechanism or not. Finally, we show the implication of our model for the measurement of peer effects, and compare and contrast them with those under the LIM and discrete choice based models. In this regard, we develop a novel equilibrium-based perspective on why the measurement of peer effects may involve nonlinearities that draws on an interaction between preferences and socially influenced attention and involves more preferred alternatives crowding out peer effects associated with less preferred ones. Additionally, we show the existence of a social multiplier.[4]

We draw on the random consideration set model of Manzini and Mariotti (2014) to capture the mutual interactions between peers' attention and choices.[5] In their choice-theoretic model, an individual decision maker has preferences over a given set of alternatives. However, her choice behavior is intermediated by the probabilities with which alternatives receive attention, which are *exogenously* specified. Given these attention probabilities, the decision maker's probability of choosing an alternative in a menu is given by the conjoint probability of the event that this alternative receives attention, but alternatives strictly preferred to it do not. The agents in our model use the same choice rule but within an equilibrium setting with the attention probabilities *endogenously* determined based on their peers' choices. Specifically, under the *attention through peers (ATP)* equilibrium solution concept we introduce, an individual's susceptibility to peer influence translates to them being more likely to pay attention to alternatives perceived as popular and more frequently chosen by their peers. An ATP equilibrium pins down the inter-connected choice behavior of these agents who mutually influence each other within their respective clusters with agents' choice probabilities dependent on attention probabilities, which in turn are dependent on these same choice probabilities. We establish that these interactions are mutually consistent and determinate by showing that an ATP equilibrium uniquely exists.

Our equilibrium analysis of peer influence and behavior connects our paper to an important body of work in the literature that has sought to study peer effects through equilibrium-based models. Prominent among these are attempts to provide micro-foundations for the

[3]See, for example, Blume et al. (2015), Boucher et al. (2024), and Brock and Durlauf (2001).

[4]A social multiplier is present when a common shock to fundamentals produces not just a direct effect on individual outcomes but also an indirect effect through social interactions, with the multiplier defined as the ratio of total direct and indirect effect to the direct effect.

[5]A similar random consideration set model also appears in Manski (1977).

linear-in-means (LIM) model of peer effects through game-theoretic models on networks in which the best response function of each agent is linear in the mean outcome/action of her peers. It is well known that this can be done through either a model of conformist social norm or one involving spillovers (Blume et al., 2015). Recently, Boucher et al. (2024) have generalized this by providing micro-foundations for non-linear aggregation over peers' outcomes, which includes as special cases the LIM model and "max" (resp., "min") model under which an agent is only influenced by the peer with the maximal (resp., minimal) outcome. When it comes to non-linearities in peer effects, Brock and Durlauf (2001) in an influential work that studies self-consistent equilibria in a binary choice model. In their set-up, individuals receive idiosyncratic random utility shocks that are logistically distributed and hold rational expectations about the distribution of their peers' actions.[6] Where our model diverges from this body of work is in its equilibrium analysis of peer effect in the context of boundedly rational agents with socially-influenced limited attention, whereas the literature cited above considers rational Bayesian agents. To the extent that the Bayesian benchmark may be particularly demanding in the context of network interactions, we see merit in pursuing such equilibrium based, boundedly rational approaches to studying peer effects. To further impress this point, we show that an ATP equilibrium can be supported as the outcome of a boundedly rational observational learning process.

Our study of peer effects in the context of a cluster network connects our work with the seminal contribution of Manski (1993) and the productive literature it has spurred around the question of identifying peer effect. In his paper, Manski discusses the reflection problem, highlighting the challenges in identifying endogenous peer effects when such peers are embedded within clusters. The interactive nature of decision making makes it challenging to discern whether the similarity in behavior among peers is due to the influence of peers' outcomes (endogenous peer effects), peers' characteristics (contextual effects), or due to shared external factors (correlated effects). Indeed, Manski (1993) establishes that even under the assumption that the cluster network is known to the econometrician, identifying endogenous peer effects may not be possible. To disentangle these effects, the literature has had to move away from cluster networks and consider peers embedded within a general network structure (Bramoullé, Djebbari, and Fortin, 2009; De Giorgi, Pellizzari, and Redaelli, 2010; Lin, 2010). In such networks, identification is possible by exploiting a novel identification strategy involving non-transitivity of peers under which peers of peers are not peers. This is, however, only one of the challenges that Manski laid out in his paper. More generally, Manski was concerned about the problem of identification of the social network itself, as this may not be observable to the econometrician.[7] Our work here shows how observed behavior can be used to address the identification challenges laid out by

[6]Other contributions that have built on this paper are Lee, Li, and Lin (2014) and Ioannides (2006).

[7]Manski (1993) writes (pg. 536): " Researchers studying social effects rarely offer empirical evidence to support their reference-group specifications. . . . If researchers do not know how individuals form reference groups and perceive reference-group outcomes, then it is reasonable to ask whether observed behavior can be used to infer these unknowns."

Manski. Not only do we fully identify the cluster network underlying peer effects, but we are also able to exactly identify individual preferences and the extent of their susceptibility to influence, which in our model underlie endogenous peer effects, thus permitting the exact identification of these effects.

At the same time, we also establish that our model is falsifiable based on choice data. That is, for any such dataset of interdependent choice behavior suggestive of peer effects, we are able to transparently determine whether it can be rationalized by the ATP mechanism or not. This behavioral characterization of the model based on three straightforward conditions holds value precisely because different micro-founded models of peer effects, based on very different mechanisms may imply identical reduced form specifications. Therefore, if our goal is to tie evidence of peer effects to precise mechanisms, we need to be able to infer about mechanisms from observed data. In the way of an example to illustrate this point, note that both a micro-founded model of conformist social norms and one of spillover or complementarities imply a linear in means specification of peer effects. However, the implications of these two models are very different, e.g., whereas the latter implies a social multiplier, the former doesn't (Boucher and Fortin, 2016). This, therefore, illustrates the importance of tying data to mechanisms and not simply to the existence of peer effects, and a behavioral characterization exercise allows us to do so.

Our approach to identification and characterization draws inspiration from the decision theory literature by exploiting inter-menu variations in behavior. In so doing, we join a recent behavioral choice theory based literature that has studied peer effects and interdependent behavior within social networks.[8] Our results also highlight how decision-theoretic tools can complement econometric approaches in solving challenging questions involving identification of networks and peer effects within them. An emerging literature in econometrics has been actively involved in measuring peer effects when the underlying network of peers is unknown to the researcher.[9] This, for instance, could be the case if either data on social ties are not available, or the collected data on such links based on common observables may not be the true links driving peer effects. In this context, our work shows how choice data and standard decision-theoretic tools may be powerful allies in aiding identification of missing network links in the data.

The final set of questions we look at in this paper is regarding the measurement of peer effects and the existence of a social multiplier. The two prominent frameworks through which peer effects have been empirically studied are the LIM model (Sacerdote, 2001; Duflo, Dupas, and Kremer, 2011; Casaburi and Reed, 2022) and discrete choice models (Brock and Durlauf, 2007; Lee, Li, and Lin, 2014; Volpe, 2025). Our analysis connects with both. Like

[8]See, for example, Chambers, Cuhadaroglu, and Masatlioglu (2023), Kashaev, Lazzati, and Xiao (2023), Borah and Kops (2018) and Cuhadaroglu (2017).

[9]See, for example, De Paula, Rasul, and Souza (2024), Lewbel, Qu, and Tang (2023) and Battaglini, Patacchini, and Rainone (2022)

with discrete choice models of peer effects, our framework implies a non-linear relationship between an individual's probability of choosing an alternative and that of her peers and, from an econometric perspective, a maximum likelihood estimation strategy allows for a consistent estimation of model parameters. We show that at the heart of this non-linearity is an interaction between individual preferences and attention, which results in a crowding out effect with more preferred alternatives crowding out the peer effects associated with less preferred ones. The structure of this non-linearity is such that when the model is written out in terms of expectations, it implies a quasi-linear relationship between an individual's expected outcome and that of her peers, with the crowding out effect determining the exact nature of the quasi-linearity. This also explains why the popular LIM model would produce biased estimates of peer effects in this environment. Although an individual's expected outcome is indeed linear in the mean expected outcome of her peers as the LIM model would predict, ignoring the non-linear crowding out effect makes LIM estimates both biased and inconsistent. Our work therefore relates to an emerging strand in the literature that has made the point that there may be significant nonlinearities involved in measuring peer effects and, accordingly, the LIM model may produce biased estimates of such effects (Boucher et al., 2024). Finally, given that influence in the ATP mechanism exhibits a spillover effect, we show that our model implies a social multiplier.

The rest of the paper is organized as follows. The next section lays out the setup of the model and defines the equilibrium notion. Section 3 shows that such an equilibrium uniquely exists and is stable. Section 4 provides our identification result, establishing that the parameters of the model are uniquely identified. Section 5 provides a behavioral characterization of the equilibrium choice data. Section 6 comments on the social multiplier and measurement of peer effects in our model and how data generated by our model squares up with common estimation techniques for peer effects. It also relate the measurement of peer effects under our model to those under the LIM and discrete choice models. Proofs of results appear in the Appendix.

2 A model of attention through peers

Let $I = \{1, \ldots, n\}$ be a set of individuals in society, with typical agents denoted by i, j, etc. These individuals populate a cluster network, i.e., the underlying undirected graph of connections is a disjoint union of complete graphs. Such a network can be conveniently specified in terms of a partition $N = \left\{N^1, ..., N^S\right\}$ of I, with each element of the partition referred to as a cluster and the partition as a clustering. Individuals within a cluster are peers who are all connected and influence each other. Individuals across clusters do not. Let $N(i)$ denote the cluster to which $i \in I$ belongs. Note that $j \in N(i)$ iff $i \in N(j)$. To keep interactions in the model interesting, we assume all clusters are non-singleton.

Let X denote a finite set of alternatives, with typical elements denoted by a, b, etc.

Each $i \in I$ has preferences over the alternatives in X, captured by a strict preference ranking $\succ_i \subseteq X \times X$.[10] We assume that the clustering exhibits some degree of preference homophily. Specifically, between any two clusters, there exist some alternatives over which the preferences of every member of one cluster agree, but this agreement is not shared by the other cluster.

Condition 2.1 (Homophily)**.** *For all pairs of clusters N^s, N^t, there exist alternatives $a, b \in X$ such that all individuals in one of the clusters prefer a to b but not in the other.*

Clusterings that satisfy this property (w.r.t. the given preferences) will be referred to as homophilous clusterings. Further, influence is meaningful only in the presence of some disagreement. Therefore, we assume that individuals in a cluster do not all have identical preferences.

With these details in place, we can present the attention channel through which peer influence works in our model. As discussed in the Introduction, this channel captures the idea that the more popular a choice is in an individual's cluster, the more likely it is to receive her attention. Formally, we model this in the following way. In any menu of alternatives $A \subseteq X, A \neq \emptyset$, we measure the popularity of any alternative $a \in A$ in individual i's cluster by the average probability with which it is chosen in her cluster, $\frac{1}{|N(i)|} \sum_{j \in N(i)} p_j(a, A)$, where $p_j(a, A)$ is the probability of i's peer j choosing $a \in A$.[11] Higher its popularity, greater the attention it receives. At the same time, not all individuals are susceptible to influence to the same extent. Let $\beta_i \in (0, 1)$ be the probability of individual i's attention being immune from influence so that $1 - \beta_i$ captures the probability of her being susceptible to influence. Accordingly, with probability β_i, the alternative receives full attention, and with complementary probability $1 - \beta_i$, the likelihood of receiving attention is equal to the average probability of this alternative being chosen in her cluster. This, therefore, implies that the probability that i pays attention to the alternative a in menu A is given by:

$$\gamma_i(a, A) := \beta_i + (1 - \beta_i) \frac{\sum_{j \in N(i)} p_j(a, A)}{|N(i)|}$$

The probability with which any alternative receives an individual's attention depends on the average probability of this alternative being chosen in her cluster. At the same time, choice probabilities themselves depend on attention probabilities. In other words, choice probabilities both determine attention probabilities and, in turn, are determined by them in an environment of mutual influence and interactions within clusters. Our equilibrium notion captures the steady state of these interactions, identifying for any given menu A, the profile $((p_i(a, A))_{a \in A})_{i \in I}$ of choice probabilities that are mutually consistent, given

[10] By a strict preference ranking, we mean a total, asymmetric and transitive binary relation.

[11] $|N(i)|$ denotes the cardinality of $N(i)$. The choice probabilities $p_j(a, A), j \in N(i)$, are, of course, endogenous.

the choice-attention interaction among peers. Note, given that attention is random, it is possible that in equilibrium there is a positive probability of an individual not paying attention to any of the available alternatives and not choosing any of them. That is, in equilibrium, $\sum_{a \in A} p_i(a, A) \leq 1$, with the inequality possibly holding strictly, which if it does, means that $1 - \sum_{a \in A} p_i(a, A)$ is the probability of i not choosing any of the available alternatives.[12]

Definition 2.1. *Given a menu A, the collection of choice probabilities $\{(p_i(a, A))_{a \in A} : \sum_{a \in A} p_i(a, A) \leq 1, i \in I\}$ is an attention through peers (ATP) equilibrium if for all $i \in I$,*

$$p_i(a, A) = \gamma_i(a, A) \prod_{b \in A : b \succ_i a} \big(1 - \gamma_i(b, A)\big) \tag{1}$$

and

$$\gamma_i(a, A) = \beta_i + (1 - \beta_i) \frac{\sum\limits_{j \in N(i)} p_j(a, A)}{|N(i)|} \tag{2}$$

An ATP equilibrium can be thought of as a steady state of the process of mutual influence that takes place within clusters. Equation 1 specifies the random consideration set rule of Manzini and Mariotti (2014), which captures individuals' probabilistic choice behavior. Under it, the probability of choosing an alternative in a menu is the probability of the event that it receives attention but alternatives preferred to it do not. That is, i's best alternative in A, say a_1, is chosen with probability $\gamma_i(a_1, A)$, while the second best alternative, say a_2, is chosen with probability $\gamma_i(a_2, A)(1 - \gamma_i(a_1, A))$, and so on. Equation 2 on the other hand specifies how peer influence within clusters determines the attention probabilities, with an alternative's likelihood of receiving attention being directly proportional to the average probability of this alternative being chosen in the cluster.

3 Existence and stability

The first obvious question that needs to be addressed about an ATP equilibrium is regarding its existence. The following result establishes that not only does such an equilibrium exists, but is also unique.

Theorem 3.1. *For any collection of strict preference rankings $\{\succ_i : i \in I\}$, homophilous clustering $N = \{N^1, \ldots, N^S\}$ of I, and immunity from influence coefficients $\{\beta_i : i \in I\}$, an ATP equilibrium exists in any menu of alternatives $A \subseteq X$. Further, this equilibrium is unique.*

[12] One interpretation of not choosing any of the alternatives in a menu is that some default alternative is chosen.

Proof: Please refer to Section A.1.

An ATP equilibrium is a static solution concept. Under it, the popularity of different choices among peers instantaneously feed into the attention of agents, with these choice and attention probabilities determined simultaneously. However, what if agents' perceptions of the popularity of alternatives, which in turn shape their attention are not formed instantaneously but with a lag? This consideration naturally raises the question of whether a less demanding dynamic process of attention formation and adjustments in behavior could make agents converge towards an ATP equilibrium. We now show that this is indeed the case and an ATP equilibrium can be provided with such a dynamic foundation.

To that end, fix some menu of alternatives $A \subseteq X$ and suppose individuals start off with some initial choice probabilities over A, $((p_i^0(a, A))_{a\in A})_{i\in I}$. To begin with, these choice probabilities of different individuals can be completely arbitrary and unrelated to one other. Assume that over a "period" of time, the popularity of alternatives in the respective clusters, as captured by the average choice probabilities based on this initial profile of individual choice probabilities, is observed by agents and shapes their attention probabilities as under the ATP mechanism. We may think of this exercise as one of agents observationally learning about the popularity of different alternatives in this menu among their peers, and this learning driving their attention. Subsequently, these attention probabilities generate updated choice probabilities according to the ATP mechanism. Suppose this process of updating attention probabilities and subsequently choice probabilities proceeds in this manner indefinitely. We may ask whether the sequence of choice probabilities generated thus converges to an ATP equilibrium. The proof of Theorem 3 allows us to answer this question in the affirmative. The proof adapts the Banach fixed-point theorem, which allows us to show that the relationship between initial and updated choice probabilities, intermediated through attention probabilities, has a contractionary property, which drives the convergence. The same argument establishes that this convergence does not depend on the initial choice of $((p_i^0(a, A))_{a\in A})_{i\in I}$.

Corollary 3.1. *For a menu $A \subseteq X$, let $p^0 = ((p_i^0(a, A))_{a\in A})_{i\in I}$ be some profile of choice probabilities. Let $\{\succ_i : i \in I\}$ be a collection of preferences, $N = \{N^1, ..., N^S\}$ a homophilous clustering, and $\{\beta_i : i \in I\}$ a collection of immunity from influence coefficients, and $p^* = ((p_i^*(a, A))_{a\in A})_{i\in I}$ the associated ATP equilibrium. Define γ^{t+1} as*

$$\gamma_i^{t+1}(a,\, A) \equiv \beta_i + (1-\beta_i) \sum_{j\in N(i)} \frac{\sum_{j\in N(i)} p_j^t(a,\, A)}{|N(i)|}$$

and correspondingly define p^{t+1} as

$$p_i^{t+1}(a,\, A) \equiv \gamma_i^{t+1}(a,\, A) \prod_{a'\in A: a'\succ_i a} (1-\gamma_i^{t+1}(a',\, A))$$

for each $i \in I$, $a \in A$. Then $\lim_{t\to\infty} p^t = p^*$.

This result, therefore, supports an interpretation of the ATP equilibrium as potentially resulting from a process of observational learning. Formally, it establishes that the equilibrium is asymptotically stable.

4 Identification

We now address the question about the identification of the parameters of the ATP model. This exercise corresponds to an important concern in the literature about the identification of structural parameters underlying peer effects. A less addressed but equally relevant question is about the identification of the set of peers itself. There is a large econometrics literature addressing the first question and an emerging one looking at the second. The approach to identification we pursue here is based on decision-theoretic tools that rely on behavioral variations across different menus of alternatives. That is, suppose an analyst happens to maintain that the data she has on a profile of choice probabilities across menus is generated by the ATP model. The question of identification is one about whether the analyst can uniquely elicit the model's parameters from this dataset. We show that for the ATP model this is indeed the case, provided the dataset is rich enough in a precise sense that we outline below.

To set this up formally, let $\mathcal{X}$ be a collection of non-empty subsets of X (i.e., menus of alternatives) over which the analyst has data on the profile of choice probabilities. A dataset is a joint random choice rule $p : \mathcal{X} \to \cup_{A\in\mathcal{X}}[0,1]^{|A|\times N}$, where for any $A \in \mathcal{X}$,

$$p(A) = \left\{ (p_i(a,\, A))_{a\in A} \,|\, \sum_{a\in A} p_i(a, A) \leq 1, i \in I \right\}$$

Let Π be the collection of all such joint random choice rules. Let Ξ be the set of all possible collections of model parameters. That is, $\xi \in \Xi$ is a tuple of preference rankings, a homophilous clustering, and immunity from influence coefficients,

$$\xi = \left(\{\succ_i : i \in I\}, N = \left\{N^1,\, ...,\, N^S\right\}, \{\beta_i : i \in I\} \right)$$

As a consequence of Theorem 3, we can define an equilibrium mapping $\mathcal{E} : \Xi \to \Pi$ for the ATP model that assigns to each $\xi \in \Xi$ a unique joint random choice rule in Π. The question of identification is essentially about a property of this equilibrium mapping.

Definition 4.1. *The parameters of the ATP model are uniquely identified if the equilibrium mapping $\mathcal{E} : \Xi \to \Pi$ is injective.*

Theorem 4.1. *If $\mathcal{X}$ contains all two alternative menus, then the parameters of the ATP model are uniquely identified.*

Proof: Please refer to Section A.3.

The key insight underlying the result is the observation that the identification problem essentially reduces to the analyst's ability to correctly identify the clusters. Once the clusters are identified, exact identification of the preferences and the immunity from influence coefficients is straightforward. To explain this in a bit more detail, suppose the analyst has succeeded in identifying the clustering $(N(i))_{i\in I}$. She can then determine for any $i \in I$ and any menu A, the average probability with which any alternative $a \in A$ is chosen in i's cluster. Denote this by $\hat{\mu}_i(a, A) = \frac{1}{N(i)} p_i(a, A)$. Clearly, as long as the clusters are uniquely identified, these average choice probabilities are as well. In the ATP model, as we establish, information about these average choice probabilities are all that is needed by the analyst to back out the preferences of the individuals and their immunity from influence coefficients. For identifying preferences, we show in Lemma A.2 and Corollary A.2 and A.3 that, for any $i \in I$ and $a, b \in X$,

$$a \succ_i b \iff \frac{1 - p_i(a, ab)}{1 - p_i(b, ab)} \leq \frac{1 - \hat{\mu}_i(a, ab)}{1 - \hat{\mu}_i(b, ab)}$$

In other words, the notion of revealed preference in the model is the following. For any individual i, and any pair of alternatives, $a, b \in X$, if the ratio of the probability of i not choosing a to not choosing b is no greater than the corresponding ratio for her cluster, then i prefers a to b. Accordingly, as soon an the analyst is able to identify any individual's cluster, she is able to identify her preferences as well.

A similar observation applies to any individual i's immunity from influence coefficient β_i. Consider any menu A in which i chooses stochastically, i.e., $p_i(b, A) \neq 1$, for any $b \in A$, and let $a = \max_{\succ_i} A$. Then, it is straightforward to derive that i's susceptibility to influence, $1 - \beta_i$ is given by,

$$1 - \beta_i = \frac{1 - p_i(a, A)}{1 - \hat{\mu}_i(a, A)}$$

The numerator, $1 - p_i(a, A)$ gives the probability of i not choosing a in A despite it being her most preferred alternative. The denominator specifies the average probability of a not being chosen in A in her cluster. Therefore, the ratio gives a measure of the extent to which a not being chosen in her cluster translates to i not choosing it, despite it being her most preferred alternative in A. Therefore, it reveals the extent of her susceptibility to influence. Accordingly, as long as an individual's cluster and her preferences can be uniquely identified by the analyst, she can also uniquely identify her immunity from influence coefficient β_i.

Therefore, the problem of uniquely identifying the parameters of the model boils down to the analyst's ability to identify the clusters. What makes such identification possible? As the reader may have guessed, a key feature of the model that enables such identification is the presence of preference homophily in the clustering. Despite the weak form in which the model assumes homophily, in the presence of random attention, it is sufficient to provide the analyst enough information to uniquely identify the clusters. To see what may go wrong w.r.t. identification when the clusterings are not homophilous, consider the following example.

Example 4.1. Suppose $X = \{a, b, c\}$ is the set of alternatives and $I = \{1, 2, 3, 4\}$ the set of individuals in society who are partitioned into the clustering $N = \{\{1, 3\}, \{2, 4\}\}$. Further let their preferences be given by: $a \succ_1 b \succ_1 c$, $a \succ_2 b \succ_2 c$, $a \succ_3 c \succ_3 b$, $a \succ_4 c \succ_4 b$; and their immunity from influence coefficients by $\beta_1 = \beta_2 = 3/5$ and $\beta_3 = \beta_4 = 2/5$. Observe that the clustering is not homophilous, as a is the common best alternative for all individuals and neither cluster agrees on ranking $\{b, c\}$. Applying the structure of interactions underlying an ATP equilibrium (equations 1 and 2 of Definition 2.1) gives us the following profile: $p_i(a, A) = 1$ for all $i \in I$ and A such that $a \in A$, and for the menu $\{b, c\}$, choice probabilities are given by,

	p_1	p_2	p_3	p_4
b	$\frac{\sqrt{1801}-31}{14}$	$\frac{\sqrt{1801}-31}{14}$	$\frac{3\sqrt{1801}-107}{28}$	$\frac{3\sqrt{1801}-107}{28}$
c	$\frac{\sqrt{1801}-4}{51}$	$\frac{\sqrt{1801}-4}{51}$	$\frac{\sqrt{1801}-21}{34}$	$\frac{\sqrt{1801}-21}{34}$

However, since individuals 1 and 2 are replicas of one another, as are 3 and 4, the clustering given by $\hat{N} = \{\{1, 4\}, \{2, 3\}\}$, along with the same preferences and influence coefficients, also represents the same profile of choice probabilities as an ATP equilibrium.

What is it that frustrates the analyst's attempt to exactly identify the clusters in this example? To answer this question, a key observation is that what permits exact identification in the model is a particular pattern of stochastic and deterministic behavior among individuals in society. Since all individuals have a well-defined preference ranking over the set of alternatives, they would in the absence of peer influence deterministically choose the best alternative in any menu according to their preferences. Hence, the presence of stochastic behavior suggests the possibility of peer influence through the random attention channel. At the same time, it is possible that even with peer influence they choose deterministically. However, for them to do so under an ATP equilibrium what needs to be true is that everyone else in their cluster does likewise (Corollary A.1). In other words, the analyst may get an "upper bound" of an individual's cluster by identifying all those who choose an alternative from a menu with probability one whenever she does so. But to get exact identification, there needs to be at least one menu where only those in her cluster choose an alternative deterministically along with her and no one outside her cluster does so. It is precisely here that homophilous clusterings comes to the analyst's aid in identification. Homophilous clusterings, by ensuring that between any two clusters there exists at least some pair of alternatives over which one cluster unanimously prefers one alternative and the other does not, guarantees that exact identification of clusters is possible. Going back to the example, we see this clearly as the menus in which individuals choose deterministically are the same for all of them. These are precisely those menus that include alternative a, which is chosen with probability one in all of them. Based on this choice data, the best that the analyst can do is to over-identify everyone's cluster as all of society. However, if the underlying clustering is not homophilous, she can't improve upon this upper bound any further.

At the same time, it should be noted that even without the homophily assumption, there are profiles of choice probabilities that are rationalized by a unique collection of model parameters. In Appendix A.5, we provide an example illustrating this point. However, at the level of the equilibrium mapping, so long as there are at least 4 individuals in society, Example 4.1 highlights the manner in which we can construct a profile of choice probabilities that can be represented by multiple clusterings. That is, the equilibrium mapping will not be injective.

Finally, the result also highlights that an analyst need not obtain choice data on all menus to be able to identify the parameters of the ATP model. Having data on all binary menus suffices for unique identification. The following example highlights the importance of binary menus in the exercise of identifying model parameters.

Example 4.2. Suppose $I = \{1, 2\}$ with both individuals in a single cluster. $X = \{a, b, c\}$ and the analyst has choice data on the menus $\mathcal{X} = \{\{a, b\}, \{a, c\}, \{a, b, c\}\}$, given by $p_i(a, ab) = p_i(a, ac) = p_i(a, X) = 1$. Then, it is straightforward to verify that the preference profile $a \succ_1 b \succ_1 c$, and $a \succ_2 c \succ_2 b$, along with *any* $\beta_i \in (0, 1)$ represent this collection of choice probabilities. While that is enough to establish that the parameters of the model are not uniquely identified, we can go a step further and note that if 1 and 2's preferences were interchanged, we would still obtain parameters that represent the dataset we have.

5 Falsifiability

We now establish that our model is falsifiable based on observable choice data. To that end, we provide conditions on this data that behaviorally characterize the model, thus providing grounds for its falsifiability. In other words, suppose an analyst has a dataset comprising of a profile of choice probabilities over some set of menus $\mathcal{X}$, captured by a joint random choice rule $p : \mathcal{X} \to \cup_{A \in \mathcal{X}} [0, 1]^{|A| \times N}$. What conditions must the dataset p satisfy for the analyst to maintain the hypothesis that it was generated by the interactions underlying the ATP model? Equivalently, when can this analyst reject this hypothesis? We provide three conditions on this dataset that answers this question.

The first key idea underlying this exercise is the following. In our set-up, each individual has a single set of well-defined preferences; hence, without any other consideration or influence, she should choose deterministically. Conversely, her choosing stochastically reveals that others may influence her behavior, given the random attention that such influence generates. That is, the constraint imposed by influence on behavior is that it prevents individuals from *independently* choosing an alternative for sure from a menu like a standard decision maker. To understand this better, suppose a is i's most preferred alternative in a menu, but her peer j has a positive probability of choosing b in that menu. This being so, she would draw i's attention towards b, and introduce a *chance* of i choosing this alternative, thus ensuring that she doesn't choose a for sure. Conversely, the only scenario under which i may end up choosing a for sure is if j doesn't draw her attention towards other alternatives through her behavior, i.e., she too chooses a for sure. Of course, the argument is symmetric. i not choosing b for sure would influence j not doing so either. This reasoning suggests a simple way to behaviorally determine who is connected to whom in the influence network. Any $i, j \in I$ are (revealed to be) *connected* if for any $A \in \mathcal{X}$ and $a \in A$, $p_i(a, A) = 1$ if and only if $p_j(a, A) = 1$. Our first condition on the dataset p requires it to be consistent with the observation that no individual is an island and everyone has connections, in addition to all such individuals exhibiting stochasticity in their behavior. Note that we say that $i \in I$ chooses stochastically if there exists some $A \in \mathcal{X}$, s.t. $p_i(a, A) \neq 1$, for any $a \in A$.

Axiom 1 (**Peer influence**). *Each $i \in I$ chooses stochastically, and for any such i there exists $j \neq i$ such that i and j are connected.*

Denote the set of individuals to whom $i \in I$ is connected by $R(i)$. That is,

$$R(i) = \{j \in I : p_i(a,\, A) = 1 \iff p_j(a,\, A) = 1, A \in \mathcal{X},\, a \in A\}$$

Further, define for any $A \in \mathcal{X}$ and $a \in A$, the average choice probability of a being chosen in A in $R(i)$ by:

$$\mu_i(a, A) = \frac{1}{|R(i)|} \sum_{j \in R(i)} p_j(a, A)$$

Our next condition delves into the question of when does a DM's behavior reveal that she has consistent preferences. Suppose, for some menu A and $a \in A$, $p_i(a, A) \geq p_i(b, A)$ or, equivalently, $\frac{1-p_i(a,A)}{1-p_i(b,A)} \leq 1$, for all $b \in B \setminus a$. From this, can we infer that a is i's most preferred alternative in A? Well, not necessarily, because the choice probabilities, $p_i(a, A)$ and $p_i(b, A)$, may reflect not just i's preference but also the relative popularity of a vs. b amongst i's connections, as captured by $\mu_i(a, A)$ and $\mu_i(b, A)$. Hence, to make a correct inference, one needs to take cognizance of this. One may do so by comparing the probability ratio of i not choosing a to not choosing b, $\frac{1-p_i(a,A)}{1-p_i(b,A)}$, to the corresponding ratio capturing average choice behavior among her connections, $\frac{1-\mu_i(a,A)}{1-\mu_i(b,A)}$. In particular, if the first ratio is no larger than the second for any $b \in A \backslash a$, then it reveals that a is indeed i's most preferred alternative in A. Our next condition draws on the standard independence of irrelevant alternatives (IIA) condition for deterministic choices to introduce the requirement that such inferences about a DM's preferences should not be menu dependent.

Axiom 2 (**Stochastic IIA**)**.** *For all $i \in I$ and $A \in \mathcal{X}$, $\exists!$ $a \in A$ such that $\frac{1-p_i(a,A)}{1-p_i(b,A)} \leq \frac{1-\mu_i(a,A)}{1-\mu_i(b,A)}, \forall b \in A \setminus a$; and for any $B \subseteq A$, with $a \in B$, $\frac{1-p_i(a,B)}{1-p_i(b,B)} \leq \frac{1-\mu_i(a,B)}{1-\mu_i(b,B)}, \forall b \in B \setminus a$.*

That is, in any menu A, there exists an alternative $a \in A$ that is revealed to be the most preferred, and in any sub-menu B of A in which a is present, the same is true.

Our final condition seeks to elicit from behavior the extent to which a DM is susceptible to influence; and it imposes an independence requirement on such idiosyncratic susceptibility to influence. To understand the idea behind the condition, note that for any menu A in which i chooses stochastically, if $a_1 \in A$ is i's most preferred alternative in the menu, then the ratio $\frac{1-p_i(a_1,A)}{1-\mu_i(a_1,A)}$ captures the extent of i's susceptibility to influence when it comes to her likelihood of choosing a_1. For instance, suppose $p_i(a_1, A) = 0.9$ and $\mu_i(a_1, A) = 0.6$. Then it means that a 40% chance of a_1 not being chosen on average amongst her connections translates into a 10% chance of this alternative not being chosen by i, even though it is her most preferred alternative. Hence, the ratio $\frac{0.1}{0.4} = \frac{1}{4}$ captures i's idiosyncratic susceptibility to influence. Now consider i's second most preferred alternative in A, call it a_2. Note that w.r.t. a_2, $\frac{1-p_i(a_2,A)}{1-\mu_i(a_2,A)}$ is not the correct measure of idiosyncratic susceptibility to influence. This is because a_2 is not i's most preferred alternative in A and that is part of the reason why this alternative may not be chosen. Therefore, the correct probability to look at is not the unconditional probability of a_2 not being chosen, $1 - p_i(a_2, A)$, but rather the probability that a_2 is not chosen conditional on no alternative preferred to it, i.e., a_1, being chosen; specifically, the probability $1 - \frac{p_i(a_2,A)}{1-p_i(a_1,A)}$. The susceptibility to influence w.r.t. the choice of a_2 is then captured by the ratio $\frac{1-\frac{p_i(a_2,A)}{1-p_i(a_1,A)}}{1-\mu_i(a_2,A)}$. If this susceptibility to influence is independent of the alternatives to which the DM pays attention, then it should be the case that $\frac{1-p_i(a_1,A)}{1-\mu_i(a_1,A)} = \frac{1-\frac{p_i(a_2,A)}{1-p_i(a_1,A)}}{1-\mu_i(a_2,A)}$. Our final condition requires that this idiosyncratic susceptibility to influence should not vary across alternatives in a menu, nor should it vary across menus. To capture this formally, we need to introduce some notation. First, for any menu A in which i chooses stochastically, i.e., $p_i(b, A) \neq 1$, for any $b \in A$, and $a \in A$, define:

$$\bar{A}_i(a) = \left\{ b \in A : \frac{1 - p_i(b, ab)}{1 - p_i(a, ab)} \leq \frac{1 - \mu_i(b, ab)}{1 - \mu_i(a, ab)} \right\}$$

Based on our discussion above, $\bar{A}_i(a)$ contains those alternatives in A that are revealed to be preferred by i to a. Then the probability of a being chosen conditional on no alternative that is revealed to be preferred to it being chosen is given by:

$$p_i^*(a, A) = \frac{p_i(a, A)}{1 - \sum_{b \in \bar{A}_i(a)} p_i(b, A)}$$

Axiom 3 (**Menu independence of influence**)**.** *For any $i \in I$, menus $A, B \in \mathcal{X}$ in* *which i chooses stochastically, and $a \in A$, $b \in B$,*

$$\frac{1 - p_i^*(a, A)}{1 - \mu_i(a, A)} = \frac{1 - p_i^*(b, B)}{1 - \mu_i(b, B)} < 1$$

These three conditions together characterize the ATP model. We say that the dataset $p : \mathcal{X} \to \cup_{A \in \mathcal{X}} [0, 1]^{|A| \times N}$ is ATP rationalizable if there exists model parameters $(\{\succ_i : i \in I\}, N = \{N^1, ..., N^S\}, \{\beta_i : i \in I\})$ such that for any menu $A \in \mathcal{X}$, $(p_i(., A))_{i \in I}$ is an ATP equilibrium for those parameters.

Theorem 5.1. *Suppose all two and three alternative menus are in $\mathcal{X}$. Then the dataset $p : \mathcal{X} \to \cup_{A \in \mathcal{X}} [0, 1]^{|A| \times N}$ is* ATP rationalizable *if and only if it satisfies peer influence, stochastic IIA, and menu independence of influence.*

Proof: Please refer to Section A.4.

Remark 5.1. Manzini and Mariotti (2014) note that in an individual choice setting, a random consideration set model with menu-dependent attention probabilities can neither be falsified, nor can the individual's preferences or attention probabilities be identified. In our model, even though attention probabilities are menu-dependent, due to their endogenous determination through peer behavior, the model is indeed falsifiable, as the result above shows. Moreover, the mechanism underlying attention probabilities allows us to uniquely identify the individual's set of peers, and thereby their preferences and immunity from influence coefficients through comparison of their choice probabilities with those of their peers. We leave it for future research to investigate under what conditions are models with menu-dependent attention probabilities falsifiable.

6 Measurement of peer effects and the social multiplier

We now study what our model implies for the measurement of peer effects and the presence of a social multiplier. The leading models through which endogenous peer effects have been measured in the literature are the linear-in-means (LIM) model and the discrete choice model. Under the LIM model, endogenous social effects are captured in terms of a stochastic linear relationship between an individual's outcome or action and the average of the same in her peer group. Discrete choice models capture this through a non-linear specification that makes an individual's probability of choosing an outcome depend on the choice probabilities of her peers. As mentioned in the Introduction, both of these specifications can be derived based on micro-founded equilibrium models of interactions. A related question that has received considerable attention is whether such interactions imply a *social multiplier*. A social multiplier exists when a common shock to fundamentals produces not just a direct effect on individual behavior but also an indirect effect through social interactions, with the multiplier defined as the ratio of the total (direct and indirect) effect to the direct effect. A key observation in the literature is that whether a social multiplier exists or not crucially depends on the underlying source of social interactions (Boucher and Fortin, 2016). For instance, both the spillover and pure conformity models of peer effects imply the linear-in-means specification, but whereas a social multiplier exists in the former, it doesn't in the latter. In other words, even though a particular set of interactions may produce an endogenous peer effect that, say, an econometrician measures from the data, it will be a mistake to assume from there that a social multiplier is also implied by those interactions.

We show below that the measurement of peer effects in our model connects both to the discrete choice and LIM set-ups. Like with discrete choice models of peer effects, it implies a non-linear relationship between an individual's choice probabilities and that of her peers. The key economic content of this non-linearity is that it implies a *crowding out* effect whereby the peer effects associated with more preferred alternatives crowds out that of less preferred ones. Further, when the model is written out in expectations form, somewhat in the spirit of the LIM model but distinct from it, it implies a *quasi-linear* relationship between own expected outcomes and average expected outcomes of peers. This quasi-linearity is a consequence of the crowding out effect, which in turn determines the measurement error that the LIM model is susceptible to. Further, interactions in the model does indeed generate a social multiplier.

6.1 Measuring endogenous peer effects

To make these observations, we consider a simple set-up in which an individual's outcome or action in some activity—for concreteness, we refer to it as effort—is determined through the ATP mechanism. Suppose individuals choose between high (e_H) and low (e_L) effort levels, with a default effort level denoted by e^*. We assume that $e_H > e_L > e^* = 0$ and consider a cluster of individuals $N^s = \{1, ..., n\}$ with associated parameters $(\succ_i)_{i \in N^s}$ and $(\beta_i)_{i \in N^s}$, where $e_H \succ_i e_L$ or $e_L \succ_i e_H$.

Consider an individual i for whom $e_H \succ_i e_L$. If the probability distribution over effort levels is generated by the ATP model, for such an individual, the probabilities of choosing each level of effort are given by:[13]

$$p_i(e) = \begin{cases} \beta_i + (1-\beta_i)\mu_H & e = e_H \\ \beta_i + (1-\beta_i)\mu_L - (1-\beta_i)\mu_L(\beta_i + (1-\beta_i)\mu_H) - \beta_i(\beta_i + (1-\beta_i)\mu_H) & e = e_L \\ (1-\beta_i)^2(1-\mu_H)(1-\mu_L) & e = e^*, \end{cases}$$

where $\mu_H = \frac{1}{n}\sum_{j \in N^s} p_j(e_H)$ and $\mu_L = \frac{1}{n}\sum_{j \in N^s} p_j(e_L)$ denote the average probability in

[13]Similar expressions can be derived for i with $e_L \succ_i e_H$.

the cluster of choosing e_H and e_L, respectively.

A key observation to understand the working of peer effects in our model is the following. Whereas it is true that the probability of exerting the most preferred level of effort, say e_H, is linear in the mean probability of peers exerting this level of effort, μ_H, the same is not true for other levels of effort. This is because the probability of choosing a particular effort level is a function of all alternatives weakly preferred to it. Since an alternative is chosen only when it receives attention and all alternatives preferred to it do not, the probability of choosing the less preferred alternative for this individual, e_L, depends not just on μ_L, but also on μ_H. Specifically, when it comes to μ_L, observe that although $p_i(e_L)$ is increasing in μ_L overall, this relationship is intermediated by both a positive and negative effect. First, there is the positive effect given by the term $(1-\beta_i)\mu_L$, according to which $p_i(e_L)$ is increasing linearly in the mean probability of peers exerting this level of effort, just like for $p_i(e_H)$. But, at the same time, there is a non-linear negative effect brought about by the interaction term $(1-\beta_i)\mu_L(\beta_i + (1-\beta_i)\mu_H)$. This term accounts for the fact that for any marginal increase in μ_L, the whole change doesn't pass on to the choice probability $p_i(e_L)$ as considering only the positive effect would suggest. Rather, only a fraction of it does, where this fraction is precisely the probability of e_H not receiving attention, i.e., the fraction $\beta_i + (1-\beta_i)\mu_H$ corresponding to the probability that e_H does receive attention needs to be discounted, and this is what the non-linear interaction term does. By this channel, therefore, the higher is μ_H, the lower is the impact of changes in μ_L on $p_i(e_L)$. This is the exact sense in which more preferred alternatives crowd out the peer effects of less preferred alternatives—the attention received by alternatives strictly preferred to it directly come in the way of higher probabilities of peers choosing this alternative translating into a decision-maker doing so. This also explains why, once this crowding out effect coming from strictly preferred alternatives are discounted, a linear-in-means relationship does indeed hold for all alternatives in a conditional sense. To see this, note that:

$$\frac{p_i(e_L)}{1-p_i(e_H)} = \beta_i + (1-\beta_i)\mu_L$$

That is, the probability of choosing e_L, conditional on no alternative preferred to it, i.e., e_H, being chosen is indeed linear in μ_L. The same relationship would also hold for a set-up with more than two alternatives.

A natural way to estimate these choice probabilities from finite observations of effort levels is to use a maximum likelihood estimator. Suppose an analyst observes m profiles of effort levels for the cluster N^s. Letting e_i^k denote the k-th observation of i's effort, the joint probability of observing the dataset $\mathbf{e} = \left(\left(e_i^k\right)_{i\in I}\right)_{1\le k\le m}$ given parameters $\theta = (\beta_i, \succ_i)_{i\in I}$ is

$$\mathbb{P}\left(\mathbf{e} \,|\, \theta\right) = \prod_{k=1}^{m}\prod_{i\in I} p_i(e_i^k)$$

where $(p_i)_{i\in I}$ is the joint random choice rule derived by the ATP mechanism given the parameters θ and the trivial clustering. Naturally, the m-adjusted log-likelihood function can be written as

$$\hat{\ell}_m(\theta|\mathbf{e}) = \sum_{i\in I}\log\left(1 - p_i(e_H) - p_i(e_L)\right) + \hat{f}_i(e_H)\log\left(\frac{p_i(e_H)}{1 - p_i(e_H) - p_i(e_L)}\right) + \sum_{i\in I}\hat{f}_i(e_L)\log\left(\frac{p_i(e_L)}{1 - p_i(e_H) - p_i(e_L)}\right)$$

where $\hat{f}_i(e)$ is the observed frequency of individual i choosing effort level e. Thus, we can observe that $\hat{f} = \left(\hat{f}_i\right)_{i\in I}$ is minimal sufficient for θ. Further, given $\hat{f}$, we can obtain the maximum likelihood estimate, $\hat{\theta}_m = \operatorname{argmax} \hat{\ell}_m(\theta\,|\,\mathbf{e})$, for the sample $\mathbf{e}$.[14] Under the assumption that preferences are not unanimous, the model parameters are uniquely identified (Theorem 4.1), and $\hat{\ell}_m(\cdot\,|\,\mathbf{e})$ is continuous in $(\beta_i)_{i\in I}$.[15] Then, by the finiteness of $-\sum_{(e_i)_{i\in I}}\left[\mathbb{P}((e_i)_{i\in I}|\theta)\sum_{i\in I}\log(p_i(e_i))\right]$ for all θ, where the summation is over all possible profiles of effort levels, $\hat{\theta}_m$ is consistent (Seo and Lindsay, 2013).

Next, we relate the measurement of peer effects in our model with that in the linear-in-means model. For that, we first write the expected effort level of an individual, given the choice probabilities derived above. For an individual for whom $e_H \succ_i e_L$, this is given by:

$$\begin{aligned}\mathbb{E}[e_i] &= p_i(e_H)e_H + p_i(e_L)e_L \\ &= \beta_i(e_H + (1-\beta_i)e_L) + (1-\beta_i)\,\mathbb{E}_\mu[e] - (1-\beta_i)e_L[\beta_i(\mu_H + \mu_L) + (1-\beta_i)\mu_H\mu_L]\end{aligned}$$

where $\mathbb{E}_\mu[e]$ is the average expected effort in i's cluster, $\frac{1}{n}\sum_{j\in N^s}\mathbb{E}[e_j]$. Equivalently, it is the expected effort based on the average probability distribution over choices in the cluster, μ. The expression highlights that the expected effort level of any individual is linear in the average expected level of her peers. But unlike a linear-in-means formulation, under our model, her expected effort also depends on the distribution of average peer behavior. This non-linear effect captured by the third term in the RHS of the expression above directly draws from the crowding out effect highlighted above.

To formally relate these insights to the LIM model, we must first accommodate the ATP specification written in expectations form into a statistical model, for which we require an exogenous source of variation in the determination of μ and subsequently $\mathbb{E}_\mu[e]$. We do so by introducing ex-ante randomness in the determination of individual preferences. For instance, suppose the preferred effort level for each individual $i \in N^s$, e_H or e_L, is determined probabilistically, independently of each other. The independent realization of preferences ensures that the only mechanism of influence between peers is through the

[14] The MLE is not well-defined if $\hat{f}_i$ is not full support for some individual i. While this may be of concern with finite observations, it becomes asymptotically irrelevant.

[15] Continuity can be established by the contraction mapping in the proof of Theorem 3 and Theorem 1 of Jachymski (2023).

choice probabilities in our model, rather than in the joint determination of preferences. Given this, under the ATP mechanism, the true relationship between expected effort of individual i and the average behavior of her peers is specified by

$$\begin{aligned}\mathbb{E}\left[e_i\right] = {} & \beta_i(e_H + (1-\beta_i)e_L) + (1-\beta_i)\,\mathbb{E}_\mu\left[e\right] \\ & - (1-\beta_i)e_L[\beta_i(\mu_H+\mu_L) + (1-\beta_i)\mu_H\mu_L] \\ & - (e_H - e_L)(\beta_i^2 + (1-\beta_i)\left[\beta_i(\mu_H+\mu_L) + (1-\beta_i)\mu_H\mu_L\right])\mathbb{1}\left[e_L \succ_i e_H\right]\end{aligned}$$

Given the distribution over preference profiles, i's expected effort, $\mathbb{E}\left[e_i\right]$ the average expected effort, $\mathbb{E}_\mu\left[e\right]$, average choice probabilities μ, and the indicator function, $\mathbb{1}\left[e_L \succ_i e_H\right]$, on preferences[16] are random variables.

Now, suppose that the data on expected effort levels underlying peer influence were being generated by the ATP model but an econometrician tried to measure peer effects using the LIM model. What is the bias that would be introduced in the measurement? In the LIM model, the behavior of any given individual is linearly influenced by the average behaviors of her peers alone. Keeping the underlying structure of the social network the same as under our model, consider the special case with simple averages of peer behavior. The LIM model would assume the expected effort of individual i to be:

$$\mathbb{E}\left[e_i\right] = \alpha_{i,\,LIM} + \gamma_{i,\,LIM}\,\mathbb{E}_\mu\left[e\right] + \epsilon_i$$

where $\alpha_{i,\,LIM}$ is a constant, and $\gamma_{i,\,LIM}$ is the peer effects coefficient under the LIM model, which measures the marginal effect of average expected outcome of peers on individual i's expected effort. Note that in the LIM model, the μ terms and indicator of preferences would be relegated to the residual. In general, these quantities are correlated with the average expected effort. As such, if choice probabilities are generated according to our model, the residual would fail to be conditionally mean-independent of the average expected effort. Then, the estimate from the LIM model, $\gamma_{i,\,LIM}$, will not be unbiased for $1-\beta_i$ from the ATP model.[17]

Even if we allow for the average probabilities of choosing e_H and e_L to be considered as linear covariates in the specification, the product term ensures that no completely linear-in-means specification can accurately account for the peer effects under our model. Finally, the sample size does not improve the accuracy of the LIM estimator. For a given cluster of size n, as the number of samples, m, increase, the sample distribution of the values of μ and $\mathbb{E}_\mu\left[e\right]$ resemble their theoretical distribution induced from the underlying distribution over preference profiles. However, given the correlation between $\mathbb{E}_\mu\left[e\right]$ and the error term, the bias in the estimation of $1-\beta_i$ persists. Thus, the estimator is inconsistent.

[16] $\mathbb{1}\left[e_L \succ_i e_H\right] = 1$, if $e_L \succ_i e_H$; 0 otherwise.

[17] Note that $Var(\mathbb{E}_\mu\left[e\right])(\gamma_{i,\,LIM} - (1-\beta_i)) = -(1-\beta_i)e_L Cov(\mathbb{E}_\mu\left[e\right],\, \beta_i(\mu_H+\mu_L) + (1-\beta_i)\mu_H\mu_L) - (e_H - e_L)(\beta_i^2 Cov(\mathbb{E}_\mu\left[e\right],\, \left[e_L \succ_i e_H\right]) + (1-\beta_i)(e_H-e_L)Cov(\mathbb{E}_\mu\left[e\right],\, \left[e_L \succ_i e_H\right]\left(\beta_i(\mu_H+\mu_L) + (1-\beta_i)\mu_H\mu_L\right))$. This is generally not zero. Numerical simulations indicate this is, in fact, positive.

6.2 The social multiplier

To analyze the question regarding the social multiplier in our model, we study the direct and indirect impact of a common shock to fundamentals on individual behavior. As an illustration, think of the common shock as the introduction of a new individual to the existing cluster. Suppose this individual prefers e_H to e_L and, to do the exercise in the simplest possible way, we assume that she is not influenced by anyone and, accordingly, chooses e_H with probability one. What we want to study is the consequence on the behavior of the existing individuals in the cluster if links are established between this individual and all of the others.

To simplify the problem, we assume that all individuals have a common immunity from influence parameter, β. Denote by k the number of individuals who prefer e_H to e_L. An ATP equilibrium can be fully characterized as a solution to a system of two equations in two unknowns, $\gamma_H(k)$ and $\gamma_L(k)$, where $\gamma_H(k)$ denotes the attention probability of e_H for an individual who prefers e_H to e_L, and $\gamma_L(k)$ denotes the the attention probability of e_L for an individual who prefers e_L to e_H:

$$\gamma_H(k) = \beta + (1-\beta)\left(\frac{k\gamma_H(k) + (n-k)(1-\gamma_L(k))\gamma_H(k)}{n}\right)$$
$$\gamma_L(k) = \beta + (1-\beta)\left(\frac{k(1-\gamma_H(k))\gamma_L(k) + (n-k)\gamma_L(k)}{n}\right)$$

To calculate the direct effect of the common shock, we consider the change to the attention probabilities from introducing this new individual who chooses e_H with probability one, while holding fixed the choice probabilities of the original individuals in the cluster. That is,

$$\gamma_H^d = \beta + (1-\beta)\left(\frac{k\gamma_H(k) + (n-k)(1-\gamma_L(k))\gamma_H(k) + 1}{n+1}\right)$$
$$\gamma_L^d = \beta + (1-\beta)\left(\frac{k(1-\gamma_L(k))\gamma_H(k) + (n-k)\gamma_L(k)}{n+1}\right)$$

Meanwhile, the total effect is calculated based on the change to the new equilibrium attention probabilities,

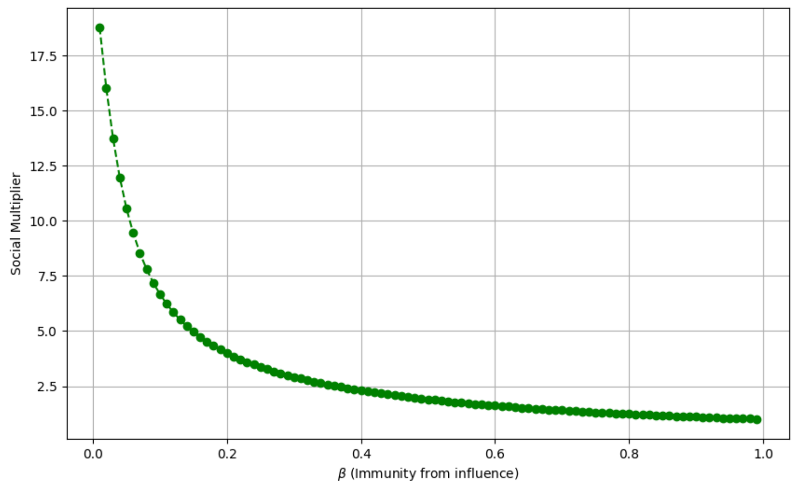
$$\gamma_H = \beta + (1-\beta)\left(\frac{k\gamma_H + (n-k)(1-\gamma_L)\gamma_H + 1}{n+1}\right)$$
$$\gamma_L = \beta + (1-\beta)\left(\frac{k(1-\gamma_H)\gamma_L + (n-k)\gamma_L}{n+1}\right)$$

We then define the social multiplier as the ratio of the change in the average expected effort between the two equilibria to the change in the average expected effort solely from the direct effect on attention probabilities,[18]

$$M = \frac{\Delta_{\text{total}}\, \mathbb{E}_\mu[e]}{\Delta_{\text{direct}}\, \mathbb{E}_\mu[e]}$$

We numerically examine the existence of the social multiplier ($M > 1$) and its relationship with k and β. For illustration, let $e_H = 2$, $e_L = 1$, $e^* = 0$, and $n = 20$. First, Figure 1 shows the relationship between the social multiplier and β with k fixed at 10. As expected,

Figure 1: Social multiplier varying with β



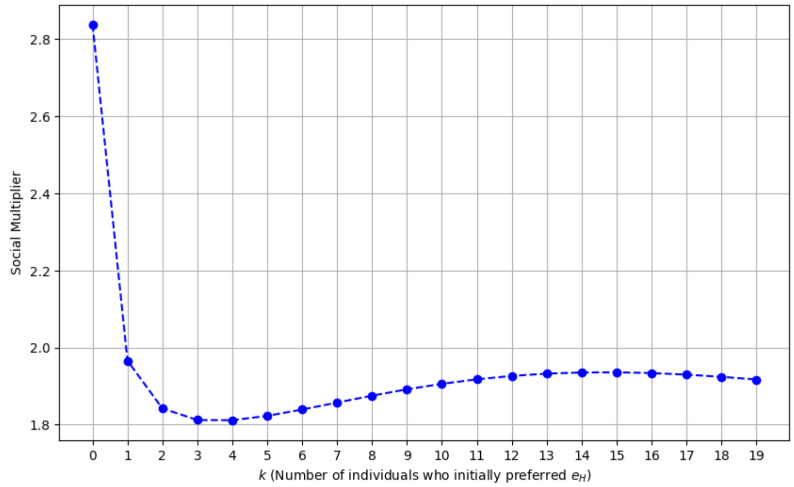
the social multiplier is inversely related to β, with the social multiplier approaching 1 as $\beta \to 1$. Since $1 - \beta$ functions as the peer effect coefficient in our model, defined as the marginal effect of an increase in average choice probability of an an alternative on an individual's choice probability, this shows that the social multiplier is indeed increasing and convex in the peer effect coefficient.

On the other hand, we see that the social multiplier varies non-monotonically with k, as we hold β fixed. Figure 2 depicts this relationship with β fixed at 0.5. This non-monotonicity of the social multiplier in k primarily arises from the non-linearity of the choice probabilities of the different effort levels. Adding a new individual who prefers e_H to e_L increases the probability of choosing e_H, but also increases the probability of choosing e^* when k is low. This effect is maximized when everyone prefers e_L, as the probability of choosing e_H suddenly increases from 0 to a positive number. However, the multiplier sharply falls thereafter, and achieves a local maximum when 14 of 20 individuals prefer e_H. Though the increase in the probability of choosing e^* from adding the new individual declines in k, so does the increase in the probability of choosing e_H. As a result, we see a declining social multiplier as k approaches n.

A key observation to make here is the existence of the social multiplier at all parameter-

[18] We exclude the newly added individual from the average

Figure 2: Social multiplier varying with k



izations in our model. As observed in the literature vis-a-vis the models of spillovers and conformity, though both imply a linear-in-means first-order condition, the former exhibits a social multiplier effect while the latter does not. This is because a positive exogenous shock to an individual's action elicits a positive response from both the individual and their peers in the presence of spillovers, but these responses are dampened under a conformity motive. In our model, peer effects are akin to those in the spillover model. An exogenous shock to an individual that increases their probability of paying attention to an alternative increases their probability of choosing that alternative, and therefore the probabilities of all their peers paying attention to said alternative. The resultant increase in the average probability of choosing the alternative further increases the attention probabilities, and so on, thus explaining the existence of the social multiplier effect.

7 Conclusion

In this paper, we have developed the micro-foundations of a theory of peer effects via the mechanism of socially influenced attention. To do so, we defined the new equilibrium notion of an attention through peers equilibrium. We established that ATP equilibrium uniquely exists for any admissible set of parameters. We then showed that the parameters of the model are uniquely identified. This exercise highlights how decision-theoretic approaches can complement econometric ones in solving challenging questions of identification in environments with social interactions and peer effects, a strategy that we hope future work shall continue to utilize in addressing questions of identification. We also provided three intuitive axioms that characterize the class of datasets that arise from ATP equilibria. This allows us to falsify the model based on choice data, which is important to do as different mechanisms of peer influence, with very different economic and policy implications may be compatible with the same reduced form specification. Finally, we related our model to the literature on the measurement of peer effects, and demonstrated how our mechanism connects to the work on estimating peer effects using discrete choice and linear-in-means models. From an econometric point of view, our model parameters can be directly estimated from choice data in a manner akin to models of discrete choice. The key point about the non-linearity in the measurement of peer effects in our model is the crowding out effect, whereby more preferred alternatives crowd out peer effects associated with less preferred ones. Furthermore, we showed that the nature of the non-linear relationship between an individual's and their peers' choice probabilities are in a manner that imply their expected outcomes are quasi-linear in the mean expected outcomes of their peers. Therefore, our model draws a meaningful connection between these two distinct strands of the literature on measuring peer effects.

While our work, in its current scope, is focused on analyzing the ATP mechanism and its theoretical implications for observable behavior, our model can be naturally extended to better suit a rigorous empirical exercise. The literature on peer effects discusses the challenges of identifying endogenous peer effects when contextual and correlated effects also affect behavior. We showed in this paper that decision-theoretic tools can be used to identify both the sets of peers and the extent of peer effects. However, we do so in an environment where the relationship between an individual's behavior and that of their peers exists solely through the endogenous peer influence channel. Incorporating contextual and correlated effects in our model would allow a full determination of the extent to which our approach complements existing empirical ones to measuring peer effects. In particular, the question of how contextual effects may impact attention appears to be an important one.[19] We hope future work shall utilize the framework we have developed here to analyze these set of questions.

A Appendix

A.1 Proof of Theorem 3.1

Note, since all interactions are intra-cluster and there are no inter-cluster interactions, it suffices to show that an ATP equilibrium exists within every cluster. Secondly, since the equilibrium notion is defined at the level of the menu, we only need to consider an arbitrary menu. Consider a cluster N^t with $|N^t| = J > 1$.

[19] For example, think of educational outcomes of students which are clearly affected by the attention students show in the classroom. Very likely, this attention depends on the classroom infrastructure and quality of the school, which are partly determined by contextual factors like the income of the neighborhood, especially when schools are financed by local taxes.

Now consider a menu $A = \{a_1, \ldots, a_M\}$. Let $\Delta_M = \{(q_1, \ldots, q_M) \in \mathbb{R}_+^M : \sum_{m=1}^M q_m \leq 1\}$ denote the unit M-simplex in $\mathbb{R}_+^M$. Define the mapping $\zeta = (\zeta_{i,m})_{i=1,\ldots,J}^{m=1,\ldots,M} : \Delta_M^J \to \Delta_M^J$ as follows: for any $i \in N^t$, $m \in \{1, \ldots, M\}$, and $p = (p_{j,m})_{j=1,\ldots,J}^{m=1,\ldots,M} \in \Delta_M^J$, let

$$\zeta_{i,m}(p) = \left(\beta_i + (1-\beta_i)\frac{\sum_{j=1}^J p_{j,m}}{J}\right) \prod_{m':a_{m'} \succ_i a_m} (1-\beta_i)\left(1 - \frac{\sum_{j=1}^J p_{j,m'}}{J}\right)$$

We first establish that ζ is a well defined mapping. To do so, establishing the following claim suffices.

<u>Claim</u>: For any $i \in N^t$ and $p \in \Delta_M^J$, $(\zeta_{i,1}(p), \zeta_{i,2}(p), \ldots, \zeta_{i,M}(p)) \in \Delta_M$.

Proof. For each $j \in N^t$ and $m = 1, \ldots, M$, $p_{j,m} \in [0,1]$ and, hence, $\mu_{i,m}(p) := \frac{\sum_{j=1}^J p_{j,m}}{J} \in [0,1]$. This in turn implies that $\gamma_{i,m}(p) := \beta_i + (1-\beta_i)\mu_{i,m}(p) \in [0,1]$ as $\beta_i \in (0,1)$; and, hence, $1 - \gamma_{i,m}(p) \in [0,1]$. Accordingly, $\zeta_{i,m}(p) = \gamma_{i,m}(p)\prod_{\{m':a_{m'} \succ_i a_m\}}(1-\gamma_{i,m'}(p))$ is a product of terms which all lie in $[0,1]$ and so $\zeta_{i,m}(p) \in [0,1]$. Further, $\zeta_{i,M+1}(p) = \prod_{m=1}^M (1-\gamma_{i,m}(p))$ is also a product of terms which all lie in $[0,1]$ and, hence, $\zeta_{i,M+1}(p) \in [0,1]$ as well.

Next, we establish that $\sum_{m=1}^M \zeta_{i,m}(p) \leq 1$. It is sufficient to establish that for each p there exists a non-negative number $z(p)$ such that $z_i(p) + \sum_{i=1}^M \zeta_{i,\,m}(p) = 1$. To that end, define $z_i : \Delta_M^J \to \mathbb{R}_+$ as

$$z_i(p) = \prod_{m=1}^M (1 - \gamma_{i,\,m}(p))$$

Since $\gamma_{i,\,m}(p) \in (0,\,1]$ for each $1 \leq m \leq M$, $z_i(p) \in [0,\,1]$. Further, assume w.l.o.g. that $a_1 \succ_i a_2 \succ_i \cdots \succ_i a_M$.

By definition, $\zeta_{i,\,M}(p) + z_i(p) = \gamma_{i,\,M}\prod_{m=1}^{M-1}(1-\gamma_{i,\,m}) + \prod_{m=1}^M(1-\gamma_{i,\,m}) = \prod_{m=1}^{M-1}(1-\gamma_{i,\,m})$.[20] Further, if $z_i(p) + \sum_{m=\ell}^M \zeta_{i,\,m}(p) = \prod_{m=1}^{\ell-1}(1-\gamma_{i,\,m})$ for some $3 \leq \ell \leq M$, then

$$\begin{aligned} z_i(p) + \sum_{m=\ell-1}^M \zeta_{i,\,m}(p) &= \zeta_{i,\,\ell-1}(p) + \prod_{m=1}^{\ell-1}(1-\gamma_{i,\,m}) \\ &= \gamma_{i,\,\ell-1}\prod_{m=1}^{\ell-2}(1-\gamma_{i,\,m}) + (1-\gamma_{i,\,\ell-1})\prod_{m=1}^{\ell-2}(1-\gamma_{i,\,m}) \\ &= \prod_{m=1}^{\ell-2}(1-\gamma_{i,\,m}) \end{aligned}$$

[20] We write $\gamma_{i,m}$ instead of $\gamma_{i,m}(p)$ to economize on notation.

Accordingly, $z_i(p) + \sum_{m=2}^{M} \zeta_{i,m}(p) = 1 - \gamma_{i,1}$ and $\zeta_{i,1}(p) = \gamma_{i,1}$, which means that $z_i(p) + \sum_{m=1}^{M} \zeta_{i,m}(p) = 1 - \gamma_{i,1} + \gamma_{i,1} = 1$. Hence, $\sum_{m=1}^{M} \zeta_{i,m}(p) \leq 1$ and $(\zeta_{i,1}(p), \ldots, \zeta_{i,M}(p)) \in \Delta_M$ and accordingly the mapping ζ is well defined. □

To show that an ATP equilibrium exists in menu A, it suffices to show that ζ has a fixed point. The uniqueness of this fixed point would yield the uniqueness of the ATP equilibrium. To that end, we use Banach's fixed point theorem.

Theorem A.1 (Banach's Fixed Point Theorem)**.** *Every contraction mapping from a complete metric space to itself admits a unique fixed point.*

$(\mathbb{R}^{M\times J}, ||\cdot||)$ is a complete metric space, where $||\cdot||$ is the Euclidean norm on $\mathbb{R}^{M\times J}$. Since Δ_M^J is a compact subset of $\mathbb{R}^{M\times J}$, it is also a complete metric space under the metric induced by the norm. Then, it is enough to show that ζ is a contraction mapping on the open set D_M^J, where $D_M = \text{int}(\Delta_M)$. To do so, write ζ as a composition of maps. First, let μ be the vector of average choice probabilities of each alternative in the cluster. That is, $\mu(m) = \frac{\sum_{j=1}^{J} p_{j,m}}{J}$. Let μ_i denote the vector that permutes the components of μ in order of $\succ_i$. That is, $\mu_{i,m}$ is the average probability of choosing the $\succ_i$ m-th best alternative in A. Then let $\nu : \Delta_M^J \to \Delta_M^J$ be a map such that its i-th component $\nu_i : \Delta_M^J \to \Delta_M$ is defined as $\nu_i(p) = \mu_i$. Now consider the operator $\overline{T} : \mathbb{R}^{M\times J} \to \mathbb{R}^{M\times J}$ that takes a vector $v \in \mathbb{R}^{M\times J}$ and is defined as $\overline{T}_{m+jM}(v) = \overline{T}_m(v) = J^{-1} \sum_{j=0}^{J-1} v_{m+jM}$. When acting on $p \in \Delta_M^J$, $\overline{T}$ gives J vertically stacked copies of μ. Since ν is a permuted version of $\overline{T}$, $|\nu| = |\overline{T}|$, where $|\cdot|$ is the operator norm, defined for a linear operator $T : \mathbb{R}^{M\times J} \to \mathbb{R}^{M\times J}$ as $|T| = \sup_{v\in\mathbb{R}^{M\times J}} \frac{||Tv||}{||v||}$. Since $\overline{T}$'s matrix under the standard basis is $J^{-1} \cdot \mathbf{1}_{J\times J} \otimes I_{M\times M}$,[21] $|\overline{T}| = 1$. Thus, $|\nu| = 1$ and ν is a non-expansive mapping.

Now, consider the mapping $\eta : \Delta_M^J \to \Delta_M^J$, such that each of its components $\eta_i : \Delta_M \to \Delta_M$ take a vector μ_i and yield a new vector of choice probabilities $\eta_i(\mu_i)$, such that applying a permutation mapping $P_i : \Delta_M \to \Delta_M$ that reorders choice probabilities in the original order of $A = \{a_1, ..., a_M\}$, to satisfy $P_i(\eta_i(\mu_i)) = \zeta_i(p)$.

To see that η is indeed a contraction mapping on Δ_M^J, we start by looking at the Jacobian matrix of η_i, $\partial\eta_i$, and start by showing that

$$\sup_{D_M} \rho(\partial\eta_i(\mu_i)) < 1$$

where $\rho(T)$ denotes the spectral radius (largest eigenvalue in absolute value) of an operator T. To do so, note that $\partial\eta_i(\mu_i)$ is a lower triangular matrix, and its eigenvalues are its diagonal entries, which are

$$\left.\frac{\partial\eta_{i,m}}{\partial\mu_{i,m}}\right|_{\mu_i} = (1-\beta_i) \prod_{m': a_{m'} \succ_i a_m} (1-\beta_i)(1-\mu_{i,m'})$$

[21] $\mathbf{1}_{J\times J}$ is a square matrix of ones, $I_{M\times M}$ is the identity matrix of size M, and $\otimes$ is the Kronecker product

Since each of the terms $(1-\beta_i)(1-\mu_{i,m'}) < 1$, $\rho(\partial\eta_i(\mu_i)) = 1-\beta_i$. Note then that $\partial\eta(\mu)$ for $\mu \in D_M^J$ is simply the block diagonal matrix with each block being $\partial\eta_i(\mu_i)$. This implies that $\partial\eta(\mu)$ is also lower triangular, and $\sup_{D_M^J} \rho(\partial\eta(\mu)) = \max_j \sup_{D_M} \rho(\partial\eta_j(\mu_j)) = 1 - \min_j \beta_j < 1$. Then, η is a locally contractive mapping on all of Δ_M^J (Hefti, 2015). Note, since Δ_M^J is compact, η is uniformly locally contractive (Jungck, 1982). Then, since Δ_M^J is convex, the shortest path between any two points in Δ_M^J is given by a straight line between them. By Lemma 2.2 of Ciesielski and Jasinski (2016), η is a contraction mapping on Δ_M^J.[22] Then, by the non-expansiveness of ν, and $P = (P_j)_{j=1}^J$ being a permutation map, $\zeta : P \circ \eta \circ \nu$ is a contraction map on Δ_M^J. Then, by the Banach fixed point theorem, ζ has a unique fixed point p^*.

A.2 Key Lemmas

We now prove a few lemmas that hold for ATP equilibria. These lemmas will be used to prove the identification result (Theorem 4.1) and the necessity of the axioms for the representation (Theorem 5.1). Let p be an ATP rationalizable joint random choice rule such that for $\xi = (\{\succ_i : i \in I\}, N, \{\beta_i : i \in I\}) \in \Xi$, $p = \mathcal{E}(\xi)$. In the way of notation, for any $i \in I$, $A \in \mathcal{X}$ and $a \in A$, let

$$\hat{\mu}_{N(i)}(a, A) = \frac{1}{|N(i)|} \sum_{j \in N(i)} p_j(a, A)$$

denote the average probability of choosing a in A in i's cluster $N(i)$. Whenever there is no ambiguity regarding the underlying clustering we are referring to when evaluating this average choice probability, we will simply write $\hat{\mu}_i$ instead of $\hat{\mu}_{N(i)}$. Further, we say that a non-singleton menu $A \in \mathcal{X}$ is a *full support menu (FSM)* for $i \in I$ if $p_i(a, A) > 0$ for all $a \in A$. We denote the set of all FSMs for i by $\mathcal{X}_i$. Then, the following conclusion follows.

Lemma A.1. *For every $i \in I$, $A \notin \mathcal{X}_i$ iff $\exists a$ such that $a = \max_{\succ_j} A$ for all $j \in N(i)$.*

Proof. Suppose there exist $j, j' \in N(i)$ such that $\max_{\succ_j} A \neq \max_{\succ_{j'}} A$. Then, if $a = \max_{\succ_i} A$, $\exists j \in N(i)$ s.t. $b = \max_{\succ_j} A$, $a \neq b$. Since $\beta_k > 0$, $\forall\, k \in I$, $p_i(a, A) = \gamma_i(a, A) = \beta_i + (1-\beta_i)\hat{\mu}_i(a, A) > 0$, and $p_j(b, A) = \gamma_j(b, A) = \beta_j + (1-\beta_j)\hat{\mu}_i(b, A) > 0$. This implies $p_i(a', A) < 1,\ \forall a' \in A \setminus a$; $p_j(a', A) < 1, \forall a' \in A \setminus b$. That is, $\forall a' \in A$, $p_\ell(a', A) < 1$, for some $\ell \in N(i)$. Accordingly, $\hat{\mu}_i(a', A) < 1,\ \forall a' \in A$.

Now consider any $c \in A$. Since $\hat{\mu}_i(c, A) < 1$ and $\beta_i \in (0, 1)$, $\gamma_i(c, A) = \beta_i + (1 -$

[22] Formally, Δ_M^J is connected, compact, and convex. As a result, it is ϵ-chainable in the language of Ciesielski and Jasinski (2016), where the $D_\epsilon(x, y)$, defined as the length of the shortest ϵ-chain from x to y is equal to $d(x, y)$ (d being the metric induced by norm $||\cdot||$ on Δ_M^J) due to convexity. As such, Δ_M^J is a contraction with respect to d.

$\beta_i)\hat{\mu}_i(c,\, A) \in (0,\, 1)$. $\gamma_i(c,\, A) \in (0,\, 1)$ implies $1 - \gamma_i(c,\, A) \in (0,\, 1)$. As a consequence, $p_i(c,\, A) = \gamma_i(c,\, A) \prod_{c' \succ_i c}(1 - \gamma_i(c',\, A)) \in (0,\, 1)$. Since this is true for all $c \in A$, $A \in \mathcal{X}_i$.

Next, suppose exists $a \in A$ such that $a = \max_{\succ_j} A$ for all $j \in N(i)$. Suppose $p_i(a, A) < 1$. Then $\hat{\mu}_i(a, A) < 1$. For all $j \in N(i)$, $\beta_j > 0$ implies

$$p_j(a, A) = \gamma_j(a, A) = \beta_j + (1 - \beta_j)\hat{\mu}_i(a, A) > \hat{\mu}_i(a, A)$$

That is, $\forall j \in N(i), p_j(a, A) > \hat{\mu}_i(a, A)$, which implies $\hat{\mu}_i(a, A) > \hat{\mu}_i(a, A)$! Hence, under an ATPE, $p_i(a,\, A) = 1$ and $A \notin \mathcal{X}_i$. □

The proof of the Lemma establishes the following corollaries.

Corollary A.1. *The following statements are equivalent:*

1. $A \notin \mathcal{X}_i$, $i \in I$
2. $p_i(a, A) = 1$*, for some* $a \in A$
3. $a = \max_{\succ_j} A$ *for all* $j \in N(i)$
4. $p_j(a, A) = 1, \forall j \in N(i)$

Lemma A.2. *Let* $A \in \mathcal{X}$ *and* $i \in I$*. Then:*

(i) $A \in \mathcal{X}_i \implies \left[a = \max_{\succ_i} A \iff \frac{1-p_i(a,A)}{1-p_i(b,A)} < \frac{1-\hat{\mu}_i(a,A)}{1-\hat{\mu}_i(b,A)},\ \forall b \in A \setminus a\right]$

(ii) $A \notin \mathcal{X}_i \implies [a = \max_{\succ_i} A \iff p_i(a, A) = 1]$

Proof. Let $A = \{a_1, \ldots, a_M\} \in \mathcal{X}_i$ be s.t. $a_1 \succ_i a_2 \succ_i \cdots \succ_i a_M$. Since $A \in \mathcal{X}_i$, for the derivation below, note that $\sum_{m=1}^{l} p_i(a_m, A) \in (0, 1)$, or equivalently, $1 - \sum_{m=1}^{l} p_i(a_m, A) \in (0, 1)$ for all $l = 1, \ldots, M-1$. Now,

$$p_i(a_1, A) = \gamma_i(a_1, A) = \beta_i + (1 - \beta_i)\hat{\mu}_i(a_1, A)$$
$$\Rightarrow 1 - \beta_i = \frac{1 - p_i(a_1, A)}{1 - \hat{\mu}_i(a_1, A)}$$

Note that, under an ATPE, an alternative $a_l \in A$ not being chosen corresponds to the event that either this alternative is not considered or some alternative preferred to it is considered. Accordingly, it is straightforward to derive that the probability that the m $\succ_i$-top alternatives are not chosen, $1 - \sum_{k \leq m} p_i(a_k,\, A)$, is equal to the probability that none of them is considered, which by the independence of the consideration of alternatives is $\prod_{k\leq m}(1-\gamma_i(a_k,\, A))$. Then,

$$
\begin{aligned}
p_i(a_m,\, A) &= \gamma_i(a_m,\, A)\prod_{k\leq m-1}(1-\gamma_i(a_k,\, A)) \\
\implies \gamma_i(a_m,\, A) &= \frac{p_i(a_m,\, A)}{1-\sum_{k\leq m-1}p_i(a_k,\, A)} \\
\implies 1-[\beta_i+(1-\beta_i)\hat{\mu}_i(a_m,\, A)] &= 1-\frac{p_i(a_m,\, A)}{1-\sum_{k\leq m-1}p_i(a_k,\, A)}
\end{aligned}
$$

Accordingly,

$$
1-\beta_i = \frac{1-\frac{p_i(a_m,\, A)}{1-\sum_{k\leq m-1}p_i(a_k,\, A)}}{1-\hat{\mu}_i(a_m,\, A)} \tag{3}
$$

Since $1-\sum_{k\leq m-1}p_i(a_k,\, A)\in(0,\, 1)$ for all $2\leq m\leq M$,

$$
\begin{aligned}
1-\frac{p_i(a_m,\, A)}{1-\sum_{k\leq m-1}p_i(a_k,\, A)} &< 1-p_i(a_m,\, A) \\
\implies \frac{1-p_i(a_1,\, A)}{1-\hat{\mu}_i(a_1,\, A)} = 1-\beta_i &< \frac{1-p_i(a_m,\, A)}{1-\hat{\mu}_i(a_m,\, A)}
\end{aligned}
$$

Hence, $\frac{1-p_i(a,A)}{1-\hat{\mu}_i(a,A)} < \frac{1-p_i(b,A)}{1-\hat{\mu}_i(b,A)}$ or $\frac{1-p_i(a,A)}{1-p_i(b,A)} < \frac{1-\hat{\mu}_i(a,A)}{1-\hat{\mu}_i(b,A)}$, $\forall b\in A\setminus a$, if $a=\max_{\succ_i}A$. To establish the only if direction, suppose $\frac{1-p_i(a,A)}{1-p_i(b,A)} < \frac{1-\hat{\mu}_i(a,A)}{1-\hat{\mu}_i(b,A)}$, $\forall b\in A\setminus a$, but $\hat{a}\neq a$ is $\succ_i$-best in A. Then based on the argument above, it follows that $\frac{1-p_i(\hat{a},A)}{1-p_i(a,A)} < \frac{1-\hat{\mu}_i(\hat{a},A)}{1-\hat{\mu}_i(a,A)}$, or $\frac{1-p_i(a,A)}{1-p_i(\hat{a},A)} > \frac{1-\hat{\mu}_i(a,A)}{1-\hat{\mu}_i(\hat{a},A)}$, a contradiction!

Now consider the case $A\notin\mathcal{X}_i$. Lemma A.1 implies that there exists $a\in A$ such that $a=\max_{\succ_j}A$ for all $j\in N(i)$. Then as shown in the proof of that Lemma and stated in Corollary A.1, $a=\max_{\succ_i}A$ iff $p_i(a,A)=1$. □

Lemma A.2 implies the following corollary:

Corollary A.2. *For all $i\in I$ and $A\in\mathcal{X}$,*

$$
a=\max_{\succ_i}A \iff \frac{1-p_i(a,A)}{1-p_i(b,A)} \leq \frac{1-\hat{\mu}_i(a,A)}{1-\hat{\mu}_i(b,A)}, \forall b\in A\setminus a
$$

Proof. If $A\in\mathcal{X}_i$, then the conclusion follows immediately from the first statement of Lemma A.2.

Now suppose $A\notin\mathcal{X}_i$. By the second statement of Lemma A.2, $a=\max_{\succ_i}A$ iff $p_i(a,\, A)=1$ iff $p_i(b,\, A)=0$, $\forall b\in A\setminus a$. Then, for any $b\in A\setminus a$, $\frac{1-p_i(a,\, A)}{1-p_i(b,\, A)}=\frac{0}{1}=0$. Further, $p_i(b,\, A)=0 \implies \hat{\mu}_i(b,\, A)<1$. Accordingly, $\frac{1-\hat{\mu}_i(a,\, A)}{1-\hat{\mu}_i(b,\, A)}$ is defined and non-negative. Thus,

$\frac{1-p_i(a,A)}{1-p_i(b,A)} \leq \frac{1-\hat{\mu}_i(a,A)}{1-\hat{\mu}_i(b,A)}, \forall b \in A \setminus a$. To establish the other direction, suppose $\frac{1-p_i(a,A)}{1-p_i(b,A)} \leq \frac{1-\hat{\mu}_i(a,A)}{1-\hat{\mu}_i(b,A)}$, $\forall\, b \in A \setminus a$, but $a' = \max_{\succ_i} A \neq a$. Then, by Lemma A.2, $1 - p_i(a', A) = 0$ and $\frac{1-p_i(a,A)}{1-p_i(a',A)}$ is undefined. Thus, the inequality cannot hold for all $b \in A \setminus a$, leading us to a contradiction. □

The following conclusion follows immediately from the last result.

Corollary A.3. *For all $i \in I$ and $a, b \in X$,*

$$a \succ_i b \iff \frac{1 - p_i(a, ab)}{1 - p_i(b, ab)} \leq \frac{1 - \hat{\mu}_i(a, ab)}{1 - \hat{\mu}_i(b, ab)}$$

A.3 Proof of Theorem 4.1

Let $\mathcal{X} \subseteq \mathcal{P}(X)$ such that $\{a, b\} \in \mathcal{X}$ for all $a, b \in X$. To show that $\mathcal{E} : \Xi \to \Pi$ is injective, we need to show that for any $p \in \Pi$ such that $p \in \mathcal{R}(\mathcal{E})$, $\mathcal{E}^{-1}(\{p\})$ is a singleton. To that end, suppose $\mathcal{E}(\xi) = \mathcal{E}(\hat{\xi})$ for $\xi = (\{\succ_i : i \in I\}, N, \{\beta_i : i \in I\})$, $\hat{\xi} = \left(\{\hat{\succ}_i : i \in I\}, \hat{N}, \left\{\hat{\beta}_i : i \in I\right\}\right) \in \Xi$. We will show that $\xi = \hat{\xi}$.

Before proving the main result, we prove an intermediate step that allows us to simplify the problem.

Lemma A.3. *Suppose $p \in \mathcal{R}(\mathcal{E})$. For all $A \in \mathcal{X}$, $p_i(a, A) = 1$ iff $p_i(a, ab) = 1$ for all $b \in A \setminus \{a\}$.*

Proof. Let $\xi = (\{\succ_i : i \in I\}, N, \{\beta_i : i \in I\}) \in \mathcal{E}^{-1}(p)$ be parameters that represent the ATP rationalizable dataset p. By Lemma A.1, $p_i(a, A) = 1$ iff $a = \max_{\succ_j} A$ for all $j \in N(i)$ iff $a \succ_j b$ for all $b \in A \setminus \{a\}$ and $j \in N(i)$. Again, by Lemma A.1, this is equivalent to $p_i(a, ab)$ for all $b \in A \setminus \{a\}$. □

For each $i \in I$, define $R(i)$ as

$$\begin{aligned} R(i) &:= \{j \in I : p_i(a, A) = 1 \iff p_j(a, A) = 1,\, A \in \mathcal{X},\, a \in A\} \\ &\equiv \{j \in I : p_i(a, ab) = 1 \iff p_j(a, ab) = 1,\, \{a, b\} \in \mathcal{X}\} \end{aligned}$$

Lemma A.3 establishes these two definitions as equivalent.

<u>**Uniqueness of Clusters**</u>: By Corollary A.1, $p_i(a, ab) = 1 \iff p_j(a, ab) = 1 \forall j \in N(i)$. Thus, $j \in N(i) \implies j \in R(i)$. Now suppose $j \notin N(i)$. Since the clustering is homophilous, there exists a pair of alternatives $\{a, b\}$ such that a is preferred to b for everyone in one of $N(i)$ and $N(j)$ but not for the other. Suppose w.l.o.g. that $a \succ_{i'} b$ for all $i' \in N(i)$. Then,

there exists $j' \in N(j)$ such that $b \succ_{j'} a$. By Corollary A.1, $p_i(a, ab) = 1$ and $p_j(a, ab) \neq 1$. Thus, $j \notin R(i)$. The same argument can be applied if $N(j)$ preferred a over b. Since $j \notin N(i) \implies j \notin R(i)$, $R(i) \subseteq N(i)$, and $N(i) = R(i)$ for all $i \in I$.

Since the same argument applies to $\hat{N}$, $N(i) = R(i) = \hat{N}(i)$ for all $i \in I$, which means that $\hat{N} = N$.

<u>**Uniqueness of Preferences**</u>: Since $N(i) = \hat{N}(i)$, $\hat{\mu}_{N(i)}(a, A) = \frac{\sum_{j\in N(i)} p_i(a, A)}{|N(i)|} = \frac{\sum_{j\in \hat{N}(i)} p_i(a, A)}{|\hat{N}(i)|} = \hat{\mu}_{\hat{N}(i)}(a, A)$ for all $a \in A$, $A \in \mathcal{X}$. Then, by Corollary A.3,

$$a \succ_i b \iff \frac{1 - p_i(a, ab)}{1 - p_i(b, ab)} \leq \frac{1 - \hat{\mu}_{N(i)}(a, ab)}{1 - \hat{\mu}_{N(i)}(b, ab)} = \frac{1 - \hat{\mu}_{\hat{N}(i)}(a, ab)}{1 - \hat{\mu}_{\hat{N}(i)}(b, ab)} \iff a \hat{\succ}_i b$$

Thus, $\succ_i = \hat{\succ}_i$ for all $i \in I$.

<u>**Uniquess of β_i**</u>: Since we assume that not everyone's preferences within a cluster are identical, for all $i \in I$, there exists $j, j' \in N(i)$ such that $a \succ_{j'} b$ but $b \succ_j a$, which holds true iff $a \hat{\succ}_{j'} b$ and $b \hat{\succ}_j a$. From Corollary A.1, $p_i(a, ab) \in (0, 1)$, which implies that $\hat{\mu}_{N(i)}(a, ab) = \hat{\mu}_{\hat{N}(i)}(a, ab) \in (0, 1)$. Putting everything together, we have

$$\begin{aligned} p_i(a, ab) &= \beta_i + (1 - \beta_i)\hat{\mu}_{N(i)}(a, ab) \\ &= \hat{\beta}_i + (1 - \hat{\beta}_i)\hat{\mu}_{\hat{N}(i)}(a, ab) \\ \implies \beta_i &= 1 - \frac{1 - p_i(a, ab)}{1 - \hat{\mu}_{N(i)}(a, ab)} = \frac{1 - p_i(a, ab)}{1 - \hat{\mu}_{\hat{N}(i)}(a, ab)} = \hat{\beta}_i \end{aligned}$$

Thus, $\beta_i = \hat{\beta}_i$ for all $i \in I$.

Thus, we have shown that $\xi = \hat{\xi}$, and $\mathcal{E}^{-1}(p)$ is a singleton for all $p \in \mathcal{R}(\mathcal{E})$. Thus, $\mathcal{E}$ is an injective mapping.

A.4 Proof of Theorem 5.1

Necessity: Suppose $p = \mathcal{E}(\xi)$ is an ATP rationalizable joint random choice rule, where ξ is a tuple of strict preference rankings $\{\succ_i: i \in I\}$, homophilous clustering $N = \{N^1, \ldots, N^S\}$, and immunity from influence coefficients $\{\beta_i : i \in I\}$. Then p satisfies:

Peer Influence: From Corollary A.1, for any menu A, $a \in A$, $p_i(a, A) = 1 \iff A$ is top-agreeable for $N(i)$ with top $a \iff p_j(a, A) = 1, \forall j \in N(i)$. Then any $j \in N(i)$ is connected to i. Since $|N(i)| > 1$, there exists $j \in N(i)$, $j \neq i$, such that j and i are connected.

Since for each cluster N^s, there exist i, j such that $\succ_i \neq \succ_j$, there exists a, $b \in X$ such that $a \succ_i b$ and $b \succ_j a$. $\{a, b\} \in \mathcal{X}$ by supposition, so $\{a, b\}$ is a menu in which individuals in N^s choose stochastically by Lemma A.1.

Stochastic IIA: We have established in the proof of Theorem 4.1 that for any ATP rationalizable choice rule with homophilous clustering $N = \{N^1, \ldots, N^S\}$, $N(i) = R(i)$, for any $i \in I$. Accordingly, $\mu_i(a, A) = \hat{\mu}_i(a, A)$, for any $A \in \mathcal{X}$. Consider any such menu A. Given that $\succ_i$ is a ranking, there exists a unique $a \in A$ such that $a = \max_{\succ_i} A$. By Corollary A.2, it follows that $\frac{1-p_i(a,A)}{1-p_i(b,A)} \leq \frac{1-\hat{\mu}_i(a,A)}{1-\hat{\mu}_i(b,A)}, \forall b \in A \setminus a$. Further, since $\hat{\mu}_i = \mu_i$, we can conclude that $\exists!\ a \in A$ such that $\frac{1-p_i(a,A)}{1-p_i(b,A)} \leq \frac{1-\mu_i(a,A)}{1-\mu_i(b,A)}, \forall b \in A \setminus a$.

Next, consider any $B \subseteq A$ with $a \in B$. Clearly $a = \max_{\succ_i} B$, and it follows from Corollary A.2 and $\hat{\mu}_i = \mu_i$ that $\frac{1-p_i(a,B)}{1-p_i(b,B)} \leq \frac{1-\mu_i(a,B)}{1-\mu_i(b,B)}, \forall b \in B \setminus a$.

Menu Independence of Influence: Let $A = \{a_1, \ldots, a_M\} \in \mathcal{X}$ be a menu in which i chooses stochastically. Suppose $a_1 \succ_i a_2 \succ_i \cdots \succ_i a_M$. Then $\bar{A}_i(a_1) = \emptyset$ and $\bar{A}_i(a_m) = \{a_1, \ldots, a_{m-1}\}$, for $m = 2, \ldots, M$. This is because by Corollary A.1 and the fact that $\mu_i(.) = \hat{\mu}_i(.)$, we know that for any $a, b \in A$, $\frac{1-p_i(a,ab)}{1-p_i(b,ab)} \leq \frac{1-\mu_i(a,ab)}{1-\mu_i(b,ab)}$ iff $a \succ_i b$. Since all binary menus are contained in $\mathcal{X}$, we can confirm the above claim. Accordingly, $p_i^*(a_1, A) = p_i(a_1, A)$, and

$$p_i^*(a_m, A) = \frac{p_i(a_m, A)}{1 - p_i(a_1, A) - p_i(a_2, A) - \cdots - p_i(a_{m-1}, A)}, \forall m = 2, \ldots, M$$

Further, drawing on equation (3) that we established in the proof of Lemma A.2 and the fact that $\mu_i(.) = \hat{\mu}_i(.)$, it follows that for any such A,

$$\frac{1 - p_i(a_1, A)}{1 - \mu_i(a_1, A)} = \frac{1 - \frac{p_i(a_2,A)}{1-p_i(a_1,A)}}{1 - \mu_i(a_2, A)} = \cdots = \frac{1 - \frac{p_i(a_M,A)}{1-p_i(a_1,A)-p_i(a_2,A)-\cdots-p_i(a_{M-1},A)}}{1 - \mu_i(a_M, A)} = 1 - \beta_i$$

i.e.,

$$\frac{1 - p_i(a_1, A)}{1 - \mu_i(a_1, A)} = \frac{1 - p_i^*(a_1, A)}{1 - \mu_i(a_1, A)} = \frac{1 - p_i^*(a_2, A)}{1 - \mu_i(a_2, A)} = \cdots = \frac{1 - p_i^*(a_M, A)}{1 - \mu_i(a_M, A)} = 1 - \beta_i < 1$$

Accordingly, for any $A, B \in \mathcal{X}$ in which i chooses stochastically, and any $a \in A$, $b \in B$, we have

$$\frac{1 - p_i^*(a, A)}{1 - \mu_i(a, A)} = \frac{1 - p_i^*(b, B)}{1 - \mu_i(b, B)} < 1$$

Sufficiency: Let p be a joint random choice rule that satisfies Peer Influence, stochastic IIA, and menu independence of influence. To show that p is ATP rationalizable we need to identify a collection of parameters $\xi \in \Xi$ such that $\mathcal{E}(\xi) = p$.

Defining the clustering: For any $i \in I$, define

$$N(i) = R(i) = \{j \in I : i \text{ and } j \text{ are connected}\}$$

We want to show that this produces a valid partition. First, note that by the symmetry in the definition, $i \in R(j) \iff j \in R(i)$. If $j \in R(i)$, then $p_{j'}(a, A) = 1 \iff p_j(a, A) = 1 \iff p_i(a, A) = 1 \iff p_{i'}(a, A) = 1$, for $j' \in R(j)$ and $i' \in R(i)$. That is, $R(i) = R(j)$. By Peer Influence, $\exists j \neq i$ s.t. $j \in R(i)$, implying that $|N(i)| > 1$.

Finally, suppose $R(i) \neq R(j)$. By the above argument, $j \notin R(i)$. Furthermore, if $j' \in R(i) \cap R(j)$, by the same argument, $R(i) = R(j') = R(j)$, which is a contradiction. Trivially, $\cup_{i \in I} R(i) = I$, which implies that $\{N(i)\}_{i \in I}$ is a valid clustering.

We next define preferences, and show at the end of the proof that this definition is consistent with N being a homophilous clustering.

Defininingpreferences: Consider $i \in I$ and define $\succ_i \subseteq X \times X$ by: for any $a, b \in X, a \neq b, a \succ_i b$ if $\frac{1-p_i(a,ab)}{1-p_i(b,ab)} \leq \frac{1-\mu_i(a,ab)}{1-\mu_i(b,ab)}$. These preferences are well-defined because $\{a, b\} \in \mathcal{X}$ for all $a, b \in X$. First, we establish that $\succ_i$ is total. This follows from stochastic IIA, since for menu $\{a, b\}$, either $\frac{1-p_i(a,ab)}{1-p_i(b,ab)} \leq \frac{1-\mu_i(a,ab)}{1-\mu_i(b,ab)}$ or $\frac{1-p_i(b,ab)}{1-p_i(a,ab)} \leq \frac{1-\mu_i(b,ab)}{1-\mu_i(a,ab)}$. Hence, we have $a \succ_i b$ or $b \succ_i a$. Since, by stochastic IIA, this inequality holds for a unique alternative, it establishes that $\succ_i$ is asymmetric. Finally, to show that $\succ_i$ is transitive, let $a \succ_i b, b \succ_i c$ and consider $A = \{a, b, c\}$. $A \in \mathcal{X}$ because $\mathcal{X}$ contains all three-alternative menus. By stochastic IIA, $\exists!\, d \in A$ such that $\frac{1-p_i(d, A)}{1-p_i(e, A)} \leq \frac{1-\mu_i(d, A)}{1-\mu_i(e, A)}$ and $\frac{1-p_i(d, de)}{1-p_i(e, de)} \leq \frac{1-\mu_i(d, de)}{1-\mu_i(e, de)}$, for all $e \in A \setminus d$. By the way $\succ_i$ is defined, given that $a \succ_i b$ and $b \succ_i c$, $d \neq b, c$. Hence, $d = a$, and consequently $a \succ_i c$.

To ensure that for each cluster N^s, there exists i, j s.t. $\succ_i \neq \succ_j$, note that there exists some a, b such that i chooses stochastically in $\{a, b\}$ by Peer Influence. We can say this about all of N^s for the same pair of alternatives by Peer Influence. Then, suppose $\frac{1-p_i(a, ab)}{1-\mu_i(a, ab)} \leq \frac{1-p_i(b, ab)}{1-\mu_i(b, ab)}$, which means that $a \succ_i b$. By stochastic IIA, this is unique to either a or b, so the inequality holds strictly for a menu in which i chooses stochastically. Since all $j \in N^s$ also choose stochastically in the menu, suppose toward a contradiction that $a \succ_j b$ for all $j \in N^s$. Then $\frac{1-p_j(a, ab)}{1-\mu_j(a, ab)} < \frac{1-p_j(b, ab)}{1-\mu_j(b, ab)}$ for all $j \in N^s$. However, this implies that $\frac{\sum_{j \in N^s} 1-p_j(a, ab)}{1-\mu_i(a, ab)} < \frac{\sum_{j \in N^s} 1-p_j(b, ab)}{1-\mu_i(b, ab)}$, which implies that $|N^s| < |N^s|$!

Defining $\beta_i's$: For any $i \in I$, define β_i as follows by taking any menu $A \in \mathcal{X}$ in which i chooses stochastically, and some $a \in A$:

$$\beta_i = \frac{p_i^*(a, A) - \mu_i(a, A)}{1 - \mu_i(a, A)}$$

Since $1 - \beta_i = \frac{1-p_i^*(a,A)}{1-\mu_i(a,A)}$, menu independence of influence guarantees that the definition of β_i is independent of the choice of a and A, and that $1 - \beta_i < 1$, or $\beta_i > 0$. By how preferences are defined, $b \succ_i a$ iff $b \in \bar{A}_i(a)$, $a, b \in A$. Then, for $a' = \max_{\succ_i} A$, since $\bar{A}_i(a') = \emptyset$, $p_i^*(a', A) = \frac{p_i(a', A)}{1-\sum_{a \in \bar{A}_i(a')} p_i(a, A)} = p_i(a', A)$. Since i chooses stochastically in A, $p_i(a', A) < 1$, which implies $\beta_i = \frac{p_i(a', A)-\mu_i(a', A)}{1-\mu_i(a', A)} < 1$.

Establishing the representation: Consider any $i \in I$. If i chooses non-stochastically in A, then there exists $a \in A$ such that $p_i(a, A) = 1 \iff p_j(a, A) = 1$ for all $j \in R(i) = N(i)$. Then:

$$\begin{aligned}\gamma_i(a, A) &=: \beta_i + (1-\beta_i)\frac{1}{|N(i)|}\sum_{j \in N(i)} p_j(a, A) \\ &= \beta_i + 1 - \beta_i \\ &= 1 \\ &= p_i(a, A)\end{aligned}$$

Note that $\frac{1-p_i(a, A)}{1-p_i(b, A)} = 0 = \frac{1-\mu_i(a, A)}{1-\mu_i(b, A)}$, for all $b \in A \setminus a$. By stochastic IIA, this implies that $\frac{1-p_i(a, ab)}{1-p_i(b, ab)} \leq \frac{1-\mu_i(a, ab)}{1-\mu_i(b, ab)}$, for all $b \in A \setminus a$, which implies by the definition of $\succ_i$ that $a = \max_{\succ_i} A$. Then, for all $b \in A \setminus a$

$$\begin{aligned}\gamma_i(b, A) \prod_{c \succ_i b;\, c \in A}(1-\gamma_i(c, A)) &= \gamma_i(b, A)(1-\gamma_i(a, A)) \prod_{c \succ_i b; c \in A \setminus a}(1-\gamma_i(c, A)) \\ &= 0 \\ &= p_i(b, A)\end{aligned}$$

Hence, the representation holds for such A.

Now, consider $A = \{a_1, ..., a_M\}$ in which i chooses stochastically, and assume wlog that $a_1 \succ_i a_2 \succ_i ... \succ_i a_M$. By the definition of $\succ_i$, $\bar{A}_i(a_1) = \emptyset$ and $\bar{A}_i(a_m) = \{a_1, ..., a_{m-1}\}$ for $m \in \{2, ..., M\}$. By menu independence of influence,

$$\beta_i = \frac{p_i^*(a_1, A) - \mu_i(a_1, A)}{1-\mu_i(a_1, A)} = \frac{p_i(a_1, A) - \mu_i(a_1, A)}{1-\mu_i(a_1, A)},$$

which implies

$$p_i(a_1, A) = \beta_i + (1-\beta_i)\mu_i(a_1, A) =: \gamma_i(a_1, A)$$

Again applying menu independence of influence, we have that for any a_m,

$$\begin{aligned}&\frac{p_i^*(a_m, A) - \mu_i(a_m, A)}{1-\mu_i(a_m, A)} = \beta_i \\ &\qquad \implies p_i^*(a_m, A) = \beta_i + (1-\beta_i)\mu_i(a_m, A) =: \gamma_i(a_m, A)\end{aligned}$$

Accordingly,

$$\frac{p_i(a_m, A)}{1-\sum_{k \leq m-1} p_i(a_k, A)} = \gamma_i(a_m, A) \tag{4}$$

Next, we establish by induction that $1 - \sum_{k \leq \ell} p_i(a_k, A) = \prod_{k \leq \ell}(1-\gamma_i(a_k, A))$. To do so, first, we know from above that $1 - p_i(a_1, A) = 1 - \gamma_i(a_1, A)$. For the inductive step, assume that the equality we want to establish holds for $\ell - 1$, i.e., $1 - \sum_{k \leq \ell-1} p_i(a_k, A) = \prod_{k \leq \ell-1}(1 - \gamma_i(a_k, A))$. Then use equation (4) to get

$$\begin{aligned} 1 - \gamma_i(a_\ell, A) &= 1 - \frac{p_i(a_\ell, A)}{1 - \sum_{k \leq \ell-1} p_i(a_k, A)} \\ &= \frac{1 - \sum_{k \leq \ell} p_i(a_k, A)}{1 - \sum_{k \leq \ell-1} p_i(a_k, A)} \\ &= \frac{1 - \sum_{k \leq \ell} p_i(a_k, A)}{\prod_{k \leq \ell-1}(1 - \gamma_i(a_k, A))} \\ \implies \prod_{k \leq \ell}(1 - \gamma_i(a_k, A)) &= 1 - \sum_{k \leq \ell} p_i(a_k, A) \end{aligned}$$

So, this expression holds for ℓ, establishing the claim. Accordingly, equation (4) implies that

$$p_i(a_m, A) = \gamma_i(a_m, A) \prod_{k \leq m-1} (1 - \gamma_i(a_k, A)),$$

for all $m \in \{2, ..., M\}$. Hence, the representation holds for all $A \in \mathcal{X}$, and $i \in I$.

Verifying that the clustering is homophilous: We know that $N(i) = R(i)$ by the definition of the clusters. Let N^s and N^t be two distinct clusters, with $i \in N^s$ and $j \in N^t$. Then, $N^s = R(i)$ and $N^t = R(j)$, with $R(i) \neq R(j)$. Since i and j are not connected, there exists a menu A such that $p_i(a, A) = 1$ or $p_j(a, A) = 1$ for some $a \in A$, but not both. Suppose w.l.o.g. that $p_i(a, A) = 1$. Then, $p_{i'}(a, A) = 1$ for all $i' \in N^s = R(i)$. In establishing the representation, we showed that $p_{i'}(a, A) = 1$ implies $a = \max_{\succ_{i'}} A$. We now want to show that $\max_{\succ_{j'}} A \neq a$ for some $j' \in R(j)$.

Choose $j' \in R(j)$ such that $p_{j'}(a, A) \leq \mu_j(a, A)$. Of course, such a j' must exist. Furthermore, $p_j(a, A) \neq 1 \implies \mu_j(a, A) < 1$. If $a = \max_{\succ_{j'}} A$, then based on the fact that choice probabilities are according to the representation as shown above,

$$\begin{aligned} p_{j'}(a, A) &= \gamma_{j'}(a, A) \\ &= \beta_i + (1 - \beta_i)\mu_j(a, A) \\ &> \mu_j(a, A) \end{aligned}$$

However, this is not true by assumption, which implies that $a \neq \max_{\succ_{j'}} A$. Then, there exists $b \in A \setminus a$ such that $b \succ_{j'} a$, even as $a \succ_{i'} b$ for all $i' \in N^s$. Therefore, the homophily condition is satisfied.

A.5 Unique identification without homophily

Example A.1. Suppose $X = \{a, b, c\}$ is the set of alternatives and $I = \{1, 2, 3, 4\}$ the set of individuals in society. Let the profile of non-trivial random choice rules for these four individuals be given by $p_i(a,\, A) = 1$ for $A = \{a, b\}, \{a, c\}, \{a, b, c\}$, $i \in I$, and for the menu $\{b,\, c\}$ choice probabilities given by the following:

	p_1	p_2	p_3	p_4
b	$1/2$	$3/5$	$2/5$	$1/2$
c	$\frac{31-\sqrt{805}}{26} \approx$ 0.101	$\frac{150-4\sqrt{805}}{481} \approx$ 0.076	$\frac{78-3\sqrt{341}}{67} \approx$ 0.337	$\frac{457-15\sqrt{341}}{1139} \approx$ 0.158

If this choice data has to be consistent with the interactions underlying an ATP equilibrium, then note first that for any menu where an individual chooses stochastically,

$$\gamma_i(a,\, A) = \beta_i + (1 - \beta_i)\hat{\mu}_i(a,\, A) > \hat{\mu}_i(a,\, A),$$

and their best alternative in the menu is chosen with the attention probability. This immediately tells us that $\{1, 4\}$ cannot form a cluster because there is no such alternative for 1 in the menu $\{b, c\}$, where her choice probability of choosing that alternative is greater than the average choice probability of 1 and 4 of choosing that alternative. By the same argument, it also follows that {1,2,3,4} cannot be part of a cluster. Note that neither $p_1(b,\, bc)$ nor $p_1(c,\, bc)$ would be greater than the averages across the four individuals.

Now look at the possibility of $\{1, 3\}$ forming a cluster. Since $p_1(b,\, bc) > p_3(b,\, bc)$ and $p_3(c,\, bc) > p_1(c,\, bc)$, we can conclude that $b \succ_1 c$ and $c \succ_3 b$ if 1 and 3 are in the same cluster. Since $p_3(b,\, bc) = 2/5$ and $p_1(b,\, bc) = 1/2$, the values for $p_1(c,\, bc)$ and $p_3(c,\, bc)$ must satisfy the following equations. The first of them is

$$\frac{p_3(c,\, bc) - \frac{1}{2}\left(p_1(c,\, bc) + p_3(c,\, bc)\right)}{1 - \frac{1}{2}(p_1(c,\, bc) + p_3(c,\, bc))} = \frac{\frac{2}{5(1-p_3(c,\, bc))} - \frac{9}{10}}{\frac{11}{20}}$$

The second is

$$\frac{2p_1(c,\, bc) - \frac{1}{2}\left(p_1(c,\, bc) + p_3(c,\, bc)\right)}{1 - \frac{1}{2}\left(p_1(c,\, bc) + p_3(c,\, bc)\right)} = \frac{\frac{1}{2} - \frac{9}{20}}{\frac{11}{20}} = \frac{1}{11}$$

The above equations come from rearranging the attention probability equations for each of the alternatives to yield an expression for β_i in terms of a menu A and some alternative $x \in A$

$$\beta_i = \frac{\gamma_i(x,\, A) - \hat{\mu}_i(x,\, A)}{1 - \hat{\mu}_i(x,\, A)}$$

Since this is equal for any $x,\, y \in A$, the above equations are obtained by equating two such expressions for β_i. It can then be verified that there is only one solution for this system of equations such that $(p_1(c,\, bc), p_3(c,\, bc)) \in [0,\, 1]^2$, which requires $p_1(c,\, bc) = \frac{457-15\sqrt{341}}{1139}$. However, this does not hold true for this profile, implying that 1 and 3 cannot be part of the same cluster either. This leaves $N = \{\{1,\, 2\}\,, \{3,\, 4\}\}$ as the only possible clustering.

Consider preferences
1: $a \succ_1 c \succ_1 b$
2: $a \succ_2 b \succ_2 c$
3: $a \succ_3 c \succ_3 b$
4: $a \succ_4 b \succ_4 c$
Then note that for values of $\beta_1 = \frac{847-29\sqrt{805}}{477+45\sqrt{805}} \approx 0.014$, $\beta_2 = 1/9$, $\beta_3 = \frac{869-36\sqrt{341}}{495+66\sqrt{341}} \approx 0.119$, $\beta_4 = 1/11$, it can be verified that $p_i(x, A) = \gamma_i(x, A) \prod_{y \succ_i x}(1 - \gamma_i(y, A))$ with $\gamma_i(x, A) = \beta_i + (1-\beta_i)\hat{\mu}_i(x, A)$. Then, this profile has an ATP representation, and by the uniqueness of the clusters, is unique.

However, note that this representation does not satisfy the weak homophily condition, as the set of top-agreeable menus for both clusters is $\{\{a, b\}, \{a, c\}, \{a, b, c\}\}$, with the tops being the same. This shows that weak homophily is not a necessary restriction on the representation for the unique identification of parameters.

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