Peer effects via socially generated attention*

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Abstract

We introduce an equilibrium-based model of peer effects in a cluster network of boundedly rational agents. The mechanism underlying peer effects draws on the idea of socially generated attention, specifically, the more popular an alternative within the network, the more likely it is to receive attention and, consequently, greater the probability of it being chosen. We introduce the solution concept of a *peer-influenced attention* equilibrium to model this choice-attention interaction among peers and establish existence. The model permits exact identification and is falsifiable: the cluster network, individual preferences, and susceptibility to influence are uniquely recovered from choice data, and three behavioral axioms characterize the mechanism. We show when a social multiplier exists, identify the type of bias that linear-in-means estimation produces, and highlight a novel preference-attention interaction that results in more preferred alternatives crowding out peer effects associated with less preferred ones.

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1 Introduction

Analyzing the influence of one's peers on behavior has been an enduring concern for economic analysis, yet the precise channels through which such peer effects operate remain a subject of debate. Indeed, it has been noted in the literature that identifying mechanisms that drive peer effects is among the more challenging open questions in this line of research and, further, that the analysis of new theoretical models of peer interactions and their subsequent estimation may be central to addressing this question (Boucher and Fortin, 2016; Bramoullé et al., 2020). The dominant view in the existing literature models peer effects either through preference interaction, with agents directly deriving utility from conforming to the actions of others, or spillover effects whereby the return that an agent

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derives from taking an action is higher if her peers do likewise. While these mechanisms capture important aspects of social behavior, they abstract from a more fundamental constraint of the modern decision-making environment: the scarcity of attention and the social determination of its rationing.

We live in an era increasingly defined as an attention economy, where the abundance of information and available alternatives vastly outstrips an individual's cognitive capacity to process them. In this environment, attention acts as a primary gatekeeper of choice, and socially generated attention has emerged as a compelling currency underwriting behavior. In such a world, peers do not merely influence how much we value an alternative; they very often determine whether we *notice* it in the first place. An alternative that is popular or trending within one's social network becomes salient, breaking through the clutter to enter the decision-maker's attention, thereby enhancing its chances of being chosen and, in turn, adding further to its popularity.

In this paper, we provide one of the first comprehensive formal analyses of this reality by theorizing a simple yet natural mechanism capturing socially generated attention and its consequence for interactive decision-making. To that end, we propose the *peer-influenced attention (PIA)* model, a tractable equilibrium-based framework of peer effects under which limited attention serves as the conduit for social influence. Unlike reduced-form approaches, we model the full feedback loop—an individual's attention is shaped by aggregate choices within her network, while those choices are, in turn, driven by the same attention constraints—with the new PIA equilibrium notion that we introduce here pinning down what mutual consistency of these peer interactions imply. Our novelty, therefore, lies in combining the behavioral phenomena of limited attention and social influence and providing a micro-foundation for an attention economy by embedding a boundedly rational model of socially generated attention within a social network. In so doing, we show that a theoretically coherent equilibrium language can be developed to talk meaningfully about this natural attention-based mechanism driving peer effects.

In our formalization, we embed social interactions within a cluster network, which has been a predominant social structure within which peer effects have been studied.[1] We then draw on the random consideration set model of Manzini and Mariotti (2014) to capture the mutual interactions between peers' attention and choices. In Manzini and Mariotti's choice-theoretic model, an individual decision maker has preferences over a

[1]Formally, the underlying graph representing the network is the disjoint union of complete graphs. Apart from the fact that such clusterings naturally form in society, one reason for this focus is that unlike diffusion where indirect connections formed between neighbors of neighbors may be adequate for propagation, the effects of influence are most prominent in direct connections: "Having a close friend engage in some behavior is likely to have more of an effect on someone than if a friend of a friend engages in that same behavior" (McAdam, 1986). Empirical evidence of peer effects in the context of a cluster network has been reported in several areas, e.g., voting behavior (Cohen, 2003; Barber and Pope, 2019; Macy et al., 2019; Miller and Conover, 2015), formative behavior like smoking, prosocial attitudes, aggressive behavior (Ehlert et al., 2020; Ellis and Zarbatany, 2017; Lodder et al., 2016), body image concerns and dieting behavior (Paxton et al., 1999), and academic performance (Zimmerman, 2003).

given set of alternatives. However, her choice behavior is intermediated by the probabilities with which alternatives receive attention, which are *exogenously* specified.[2] Given these attention probabilities, the decision maker's probability of choosing an alternative in a menu is given by the conjoint probability of the event that this alternative receives attention, but alternatives strictly preferred to it do not. The agents in our model use the same choice rule but within an equilibrium setting with the attention probabilities *endogenously* determined based on aggregate choices within their cluster, with more popular alternatives more likely to receive attention. In our baseline model, attention probabilities are linear in the average choice probabilities within a cluster. However, we also consider robustness of our results to a general non-parametric formulation of peer-influenced attention.

Our equilibrium analysis of peer influence and behavior connects our paper to an important body of work in the literature that has sought to study peer effects through equilibrium-based models. Prominent among these are attempts to provide micro-foundations for the linear-in-means (LIM) model of peer effects through game-theoretic models on networks in which the best response function of each agent is linear in the mean outcome/action of her peers. It is well known that this can be done through either a model of conformist social norm or one involving spillovers (Blume et al., 2015). Recently, Boucher et al. (2024) have generalized this by providing micro-foundations for non-linear aggregation over peers' outcomes, which includes as special cases the LIM model and "max" (resp., "min") model under which an agent is only influenced by the peer with the maximal (resp., minimal) outcome. Non-linearities in peer effects have also been studied in the context of stochastic choice models. Brock and Durlauf (2001) is a pioneering work in this line of research. They study self-consistent equilibria in a binary choice model. In their set-up, individuals receive idiosyncratic random utility shocks that are logistically distributed and hold rational expectations about the distribution of their peers' actions.[3] However, none of these micro-founded models consider the question of socially generated attention and how that may be a mechanism underlying peer effects.

At a conceptual level, a key feature that distinguishes our mechanism from this body of work is in its equilibrium analysis of peer effect in the context of boundedly rational agents with socially-influenced limited attention, whereas the literature cited above considers rational Bayesian agents. To the extent that the Bayesian benchmark may be particularly demanding in the context of network interactions, we see merit in pursuing

[2] The observation that, in any given choice problem, a boundedly rational decision maker's attention may be limited to only a subset of the available alternatives is a well-modeled behavioral phenomenon in the literature, e.g., Masatlioglu et al. (2012), Eliaz and Spiegler (2011), Manzini and Mariotti (2014), Brady and Rehbeck (2016), Lleras et al. (2017), Caplin et al. (2018), and Dardanoni et al. (2020), among others. More recently, a strand of the literature has explored the possibility that social influence may play a role in determining a decision maker's attention in the sense that she may only consider alternatives that her peers choose or they recommend, e.g., Borah and Kops (2018) and Kashaev et al. (2023).

[3] Other contributions that have built on this paper are Lee et al. (2014) and Ioannides (2006).

such equilibrium based, boundedly rational approaches to studying peer effects. Further, the way preferences and endogenous attention interact in our model produces novel non-linearities in the working of peer effects that have been missed in the existing literature. Specifically, we uncover a *crowding out effect* wherein the attention garnered by more preferred alternatives drives out some of the impact of the attention received by less preferred ones. This makes peer effects outcome specific, a possibility that the existing literature has not focused on. Therefore, even as peer effects work linearly through the mean behavior of peers, the non-linearity introduced through the distribution of peers' preferences can have important empirical and policy implications.

A central question we take up is the identification of the model's primitives from observed behavior. Our innovation in this regard is to draw on methods from the decision theory literature and rely on inter-menu variations in behavior for identification. We show, drawing on this methodology, that the model's parameters are uniquely recoverable from behavior. Although our methods are different, the motivation behind this exercise relates our work to questions in the networks econometrics literature. As is well known, Manski (1993), in a seminal contribution, brought attention to the reflection problem highlighting the challenges in identifying endogenous peer effects when such peers are embedded within clusters. Indeed, Manski's paper established that even under the assumption that the cluster network is known to the econometrician, identifying endogenous peer effects may not be possible. More generally, Manski was concerned about the problem of identification of the social network itself, as this may not be known to the econometrician.[4] Our work here suggests one strategy for how observed behavior can be used to address questions of identification in the context of peer effects. In the context of our model, we fully identify the cluster network, individuals' preferences, and the extent of their susceptibility to influence—three primitives that jointly underlie endogenous peer effects.

At the same time, we also establish that the PIA model is falsifiable by providing a characterization of the model based on choice data. This behavioral characterization, based on three straightforward axioms, provides testable restrictions that allow us to tie evidence of peer effects directly to the working of the PIA mechanism. The first axiom elicits the presence of peer influence in individual behavior. The second axiom establishes restrictions on the data that reveal consistent individual preferences. The third axiom restricts revealed individual susceptibilities to influence to be independent of the alternative under consideration. Such restrictions can prove valuable in inferring the working of precise peer influence mechanisms, especially given that models of peer effects with very different underlying mechanisms can generate identical reduced-form relationships

[4] Manski (1993) writes (pg. 536): " Researchers studying social effects rarely offer empirical evidence to support their reference-group specifications. . . . If researchers do not know how individuals form reference groups and perceive reference-group outcomes, then *it is reasonable to ask whether observed behavior can be used to infer these unknowns*" [emphasis added].

between endogenously determined observables in equilibrium. In the way of an example to illustrate this point, note that both a micro-founded model of conformist social norms and one of spillover or complementarities imply a linear in means specification of peer effects. However, the implications of these two models are very different, e.g., whereas the latter implies a social multiplier, the former doesn't (Boucher and Fortin, 2016).[5]

Our approach to identification and characterization draws inspiration from the decision theory literature by exploiting inter-menu variations in behavior. In so doing, we join a recent behavioral choice theory based literature that has studied peer effects and interdependent behavior within social networks using similar methods, including the question of identifying network links from behavior.[6] An emerging literature in econometrics has also been actively involved in measuring peer effects when the underlying network of peers is unknown to the researcher.[7] This, for instance, could be the case if either data on social ties are not available, or the collected data on such links based on common observables may not be the true links driving peer effects. In this context, our work suggests that choice data and standard decision-theoretic tools can potentially complement econometric approaches in solving challenging questions involving identification of missing network links in the data.

Our analysis abstracts from several features that would be important in a full-fledged empirical account of the attention channel generating peer effects. We do not incorporate contextual or correlated effects, or other shocks generating additional, empirically meaningful, variations in equilibrium outcomes. Our focus instead is on a thorough theorizing and analysis of the endogenous effects generated when such a mechanism is at play. Even then, our work has lessons for the applied economist looking at these questions. The final set of issues we explore in this paper speaks to this. The two prominent frameworks through which peer effects have been empirically studied are the LIM model (Sacerdote, 2001; Duflo et al., 2011; Casaburi and Reed, 2022) and discrete choice models (Brock and Durlauf, 2007; Lee et al., 2014; Volpe, 2025). Our model connects with both, as it describes a non-linear relationship between peers' choice probabilities but a quasi-linear relationship between their expectations: linear in the average cluster-level expected outcome, but non-linear in other distributional features. This non-linearity, which is a consequence of the interaction between attention and preferences and produces the crowding-out effect, is precisely why an LIM estimation strategy would produce biased estimates of peer effects in this context. We show that this bias may go in either direction, and identify conditions for the same. Our work, therefore, connects with an

[5]A social multiplier is present when a common shock to fundamentals produces not just a direct effect on individual outcomes but a reinforcing indirect effect through social interactions, with the multiplier defined as the ratio of total direct and indirect effect to the direct effect.

[6]See, for example, Chambers et al. (2023), Kashaev et al. (2023), Borah and Kops (2018) and Cuhadaroglu (2017).

[7]See, for example, De Paula et al. (2024), Lewbel et al. (2023) and Battaglini et al. (2022).

emerging literature highlighting why LIM estimation strategies may be inadequate (e.g., Boucher et al. (2024)), and we provide a novel attention-preference interaction channel for the same. We also tackle the question about the existence of a social multiplier in the context of socially generated attention. We show that the answer is nuanced and may depend on the outcome variable we are measuring, and specific values of the parameters. We also reiterate the known but sometimes ignored observation that the social multiplier is about the mechanism and a distinct consideration from the presence of positive peer effects, with no requirement that the former mirrors the latter. Indeed, as we show, the two may move in opposite directions.

The rest of the paper is organized as follows. The next section lays out the setup and defines a PIA equilibrium. Section 3 shows that such an equilibrium exists and addresses the question about its uniqueness. Section 4 provides our identification result, establishing that the parameters of the model are uniquely identified. Section 5 generalizes the attention function to allow for non-linearities and looks at what implications of a linear attention function are robust to such a change. Section 6 provides a behavioral characterization of the model. Section 7 comments on the empirical implications of our model and the social multiplier. Proofs of results appear in the Appendix.

2 A model of socially generated attention

Let $I = \{1, \ldots, n\}$ be a set of individuals in society, with typical agents denoted by i, j, etc. These individuals populate a cluster network, i.e., the underlying undirected graph of connections is a disjoint union of complete graphs. Such a network can be conveniently specified in terms of a partition $N = \left\{N^1, \ldots, N^S\right\}$ of I, with each element of the partition referred to as a cluster and the partition as a clustering. Individuals within a cluster are peers who are all connected and influence each other. Individuals across clusters do not. Let $N(i)$ denote the cluster to which $i \in I$ belongs. Note that $j \in N(i)$ iff $i \in N(j)$. To keep interactions in the model interesting, we assume all clusters are non-singleton.

Let X denote a finite set of alternatives, with typical elements denoted by a, b, etc. Each $i \in I$ has preferences over the alternatives in X, captured by a strict preference ranking $\succ_i \subseteq X \times X$.[8] We assume that the clustering exhibits some degree of preference homophily. Specifically, between any two clusters there exist alternatives over which the preferences of all members of one cluster agree, while at least some members of the other cluster disagree with that ranking.

Assumption 2.1 (Homophily)**.** *For all pairs of clusters N^s, N^t, there exist alternatives $a, b \in X$ such that all individuals in one of the clusters prefer a to b but not in the other.*

Clusterings that satisfy this property (w.r.t. the given preferences) will be referred to as homophilous clusterings. Further, influence is meaningful only in the presence of

[8]By a strict preference ranking, we mean a total, asymmetric and transitive binary relation.

some disagreement. Therefore, we assume that individuals in a cluster do not all have identical preferences.

Behavior in the model is determined not just by preferences but also by peer-influenced attention, which may be random. Accordingly, an individual's choices may be stochastic. For any such individual $j \in I$, menu of alternatives $A \subseteq X, A \neq \emptyset$, and $a \in A$, $p_j(a, A)$ will denote the probability of j choosing a in the menu A.

With these details in place, we can present the attention channel through which influence works in our model. As discussed in the Introduction, this channel captures the idea that the more popular a choice is within a cluster, the more likely it is to receive attention. Formally, we model this in the following way. In any menu of alternatives A, we measure the popularity of any alternative $a \in A$ in cluster N^s by the average probability with which it is chosen in that cluster, $\frac{1}{|N^s|} \sum_{j \in N^s} p_j(a, A)$.[9] Think of this average as a summary statistic capturing perceptions about the popularity of this alternative within the cluster, with greater popularity translating to greater attention.

At the same time, not all individuals are susceptible to influence to the same extent. Let $\beta_i \in (0, 1)$ be the probability with which individual i's attention is immune from influence so that $1 - \beta_i$ captures the probability of her being susceptible to influence. Accordingly, for any alternative $a \in A$, with probability β_i, this alternative receives full attention, and with complementary probability $1 - \beta_i$, the likelihood of it receiving attention is equal to the average probability with which it is chosen in her cluster, i.e., its popularity.[10] This, therefore, implies that the probability with which i pays attention to alternative a in menu A is given by:[11]

$$\gamma_i(a, A) := \beta_i + (1 - \beta_i) \frac{\sum_{j \in N(i)} p_j(a, A)}{|N(i)|} \tag{1}$$

With the process determining attention probabilities specified, we need to address the other side of the model pertaining to how these attention probabilities enter an individual's choice probabilities. We model the exact form of the dependence of choice probabilities on attention probabilities and preferences based on the random consideration set rule of Manzini and Mariotti (2014). Drawing on this rule, first, whether an alternative $a \in A$ receives an individual i's attention in menu A is determined by the

[9] $|N(i)|$ denotes the cardinality of $N(i)$.

[10] We assume here that β_i is independent of the alternative under consideration. While we acknowledge that the likelihood of being susceptible to influence may be alternative specific, we maintain this assumption here in the interest of model discipline and placing meaningful restrictions on the data that the model generates.

[11] The linear formulation of the attention probability function with its compound lottery interpretation and tractable theoretical structure is a natural starting point for modeling socially generated attention. In the subsequent analysis, as a robustness exercise, we also discuss general non-parametric formulations of the attention process and answer the question about whether our conclusions carry over to such formulations.

probability $\gamma_i(a, A)$ derived above, with these determinations being independent across alternatives. Then, from the realized set of alternatives that receive attention, she chooses her *most preferred* alternative. Intuitively, if a, b and c are the alternatives that receive her attention ex post and $a \succ_i b \succ_i c$, then it is only natural that she would pick a over b and c. Accordingly, the individual's ex ante probability of choosing an alternative is simply the probability of the event that it receives attention but alternatives preferred to it do not. That is, for any $a \in A$,

$$p_i(a, A) = \gamma_i(a, A) \prod_{b \in A: b \succ_i a} \big(1 - \gamma_i(b, A)\big) \tag{2}$$

Putting the two sides together, we see that choice probabilities both determine and are determined by attention probabilities within a process of influence within clusters. Our solution concept of a *peer-influenced attention (PIA)* equilibrium pins down the set of choice probability profiles that simultaneously satisfy these choice-attention interactions within the clusters. Before we formally define it, note that given attention is random, it is possible that in equilibrium there is a positive probability of an individual not paying attention to any of the available alternatives and not choosing any of them. That is, in equilibrium, $\sum_{a \in A} p_i(a, A) \leq 1$, with the inequality possibly holding strictly, which if it does, means that $1 - \sum_{a \in A} p_i(a, A)$ is the probability of i not choosing any of the available alternatives.[12]

Definition 2.1. *Given a menu A, the collection of choice probabilities $\{(p_i(a, A))_{a \in A} : \sum_{a \in A} p_i(a, A) \leq 1, i \in I\}$ is a peer-influenced attention (PIA) equilibrium if for all $i \in I$ and $a \in A$,*

$$p_i(a, A) = \gamma_i(a, A) \prod_{b \in A: b \succ_i a} \big(1 - \gamma_i(b, A)\big)$$

and

$$\gamma_i(a, A) = \beta_i + (1 - \beta_i) \frac{\sum\limits_{j \in N(i)} p_j(a, A)}{|N(i)|}$$

Interaction between preferences and socially generated attention

Equilibrium behavior under the PIA model is determined by the joint working of preferences and endogenously generated social attention. The interaction of the two produces important implications for the model. First, note that the impact that socially generated attention has on behavior is intermediated by the immunity from influence coefficient. For any individual $i \in I$, the lower the coefficient β_i, the higher the relative impact of socially generated attention on behavior and lower that of her preferences. Indeed,

[12] One interpretation of not choosing any of the alternatives in a menu is that some default alternative is chosen instead.

when β_i is low enough, choice probabilities may not agree, even directionally, with preferences. Conversely, for attention not to override preferences, β_i has to be sufficiently high. The following result provides a sufficient condition for an individual's choice probabilities under a PIA equilibrium to be in line with her preferences.

Proposition 2.1. *For any $i \in I$, if $\beta_i \geq \frac{1}{2}$, then for any menu $A \subseteq X$ and $a, b \in A$, $a \succ_i b \implies p_i(a, A) \geq p_i(b, A)$, holding strictly whenever $p_i(a, A) > 0$.*[13]

Proof: Please refer to Appendix A.2

The interaction between preferences and attention in the model is non-separable and has an important economic content for the way peer effects, understood as the relationship between an individual's choice probabilities and that of her peers, work. This is because the consequence of the attention mechanism on behavior mediates through preferences and produces an asymmetry between the peer effects associated with more preferred alternatives and less preferred ones. Specifically, the socially generated attention garnered by alternatives that an individual prefers more can dampen the impact of such attention that less preferred alternatives receive. To appreciate this crucial point, consider a simple set-up in which an individual's outcome or action in some activity—for concreteness, an effort choice—is determined through the PIA mechanism. Suppose individuals choose between high (e_H) and low (e_L) effort levels, with a default effort level denoted by e^*. We assume that $e_H > e_L > e^* = 0$. For the purpose of this example, which appears later in the analysis as well, we assume that society is organized in a single cluster $N^1 = \{1, ..., n\}$ with associated parameters $(\succ_i)_{i \in N^1}$ and $(\beta_i)_{i \in N^1}$, where $e_H \succ_i e_L$ or $e_L \succ_i e_H$.

Consider an individual i for whom $e_H \succ_i e_L$. If the probability distribution over effort levels is generated by the PIA model, for such an individual, the probabilities of choosing each level of effort are given by:[14]

$$p_i(e) = \begin{cases} \beta_i + (1-\beta_i)\mu_H & e = e_H \\ \beta_i + (1-\beta_i)\mu_L - (1-\beta_i)\mu_L(\beta_i + (1-\beta_i)\mu_H) - \beta_i(\beta_i + (1-\beta_i)\mu_H) & e = e_L \\ (1-\beta_i)^2(1-\mu_H)(1-\mu_L) & e = e^*, \end{cases}$$

where $\mu_H = \frac{1}{n}\sum_{j \in N^1} p_j(e_H)$ and $\mu_L = \frac{1}{n}\sum_{j \in N^1} p_j(e_L)$ denote the average probability in the cluster of choosing e_H and e_L, respectively.[15]

The above expressions clarify that whereas it is true that the probability of an individual exerting her most preferred level of effort, say e_H, is linear in μ_H, the mean

[13] Of course, it follows from the result that whenever $\beta_i \geq \frac{1}{2}$, if $p_i(a, A) > p_i(b, A)$ then $a \succ_i b$.

[14] Similar expressions can be derived for i with $e_L \succ_i e_H$.

[15] Peer effects equations like the ones above can be written with the average specified in an "include-i" or "exclude-i" form. Both versions are used in the literature and it is straightforward to go from one to the other. We adopt the former for this discussion. The content of our conclusions, of course, wouldn't change under the latter.

probability of this effort choice in her cluster, the same is not true for other levels of effort. Since an alternative is chosen only when it receives attention and all alternatives preferred to it do not, the probability of choosing the less preferred alternative for this individual, e_L, depends not just on μ_L, but also on μ_H. Specifically, when it comes to μ_L, observe that although there is a positive relationship between $p_i(e_L)$ and μ_L, this relationship is intermediated by both a positive and negative effect. First, there is the positive effect given by the term $(1-\beta_i)\mu_L$, according to which $p_i(e_L)$ is linear in the mean probability of this effort choice in the cluster, just like for $p_i(e_H)$. However, there is also a non-linear negative effect whereby a fraction $\beta_i + (1-\beta_i)\mu_H$ of the first effect gets discounted, where this fraction corresponds to the probability of the preferred alternative e_H receiving attention. This is because in the event that both e_H and e_L receive attention ex post, e_H gets chosen since it is preferred according to preferences, and the socially generated attention garnered by the cluster-level popularity of e_L has no impact on behavior. Accordingly, ex ante, this translates into a more subdued relationship between μ_L and $p_i(e_L)$, compared to the corresponding one for e_H. This is the sense in which there is a preference-mediated asymmetry in the way peer effects operate in the model. The attention garnered by more preferred alternatives drives out some of the potential impact of the popularity and resulting attention surrounding less preferred alternatives, dampening the peer effects associated with them. One may think of this as a *crowding out* effect at play in the way socially generated attention translates into behavioral impacts. This also explains why, once this effect coming from preferred alternatives is accounted for, a linear-in-means relationship does indeed hold for all alternatives in a conditional sense. To see this, note that:

$$\frac{p_i(e_L)}{1-p_i(e_H)} = \beta_i + (1-\beta_i)\mu_L$$

That is, the probability of choosing e_L, conditional on no alternative preferred to it, i.e., e_H, being chosen is indeed linear in μ_L. The same relationship would also hold for a set-up with more than two alternatives.

As we show in greater detail in Section 7, the crowding out effect implies that the distribution of preferences in a cluster plays a crucial role in determining the scope of endogenous effects in the model. For instance, think of an intervention that exogenously increases the attention garnered by e_H, say, by introducing an exemplary student who is not influenced and chooses e_H with probability 1. Such an intervention will have very different implications in case the overwhelming majority of students in the cluster prefer e_L compared to when a majority prefer e_H. In the former case, a much greater portion of the exogenously generated attention garnered by e_H will get crowded out and change in behavior will be much more modest.

3 Existence and uniqueness of PIA equilibrium

3.1 Existence

The next question that needs to be addressed about a PIA equilibrium is regarding its existence. The following result establishes that such an equilibrium exists for all specifications of model parameters. Since the rest of the discussion in this section uses the objects introduced in the proof of this result, we include it in the body.

Theorem 3.1. *For any collection of strict preference rankings $\{\succ_i : i \in I\}$, homophilous clustering $N = \{N^1, \ldots, N^S\}$ of I, and immunity from influence coefficients $\{\beta_i : i \in I\}$, a PIA equilibrium exists in any menu of alternatives $A \subseteq X$.*

Proof: Since all interactions are intra-cluster, in proving existence, it suffices to show that a PIA profile exists within every cluster. Let $A = \{a_1, ..., a_M\}$ be a menu of alternatives, and N^s be a cluster with J agents. Let $(\beta_i)_{i\in N^s}$ and $(\succ_i)_{i\in N^s}$ be the set of immunity from influence coefficients and preferences of the agents in N^s.

Case 1: $\exists\ a \in A$ that is the $\succ_i$-best alternative in A for all $i \in N^s$

Observe that $p_i(a, A) = 1$ and $p_i(a', A) = 0$ for all other $a' \neq a$ defines a PIA equilibrium. By Lemma A.1, this is also the unique PIA equilibrium.

Case 2: For some $a, b \in A$, $a \neq b$, $\exists i_a, i_b \in N^s$ such that $a = \max_{\succ_{i_a}} A$ and $b = \max_{\succ_{i_b}} A$

We show that the profile of random choice rules $(p_i)_{i\in N^s}$ is a PIA equilibrium if and only if the vector of average choice probabilities $\mu \in \Delta_M$ is consistent with a PIA equilibrium, where $\Delta_M = \left\{\mu \in \mathbb{R}^M_+ : \sum_m \mu_m \leq 1\right\}$. Any profile of random choice rules that is a PIA equilibrium must be generated from average choice probabilities using the PIA mechanism.[16] Particularly, define $\Gamma : \Delta_M \to [0, 1]^{JM}$ such that:

$$\Gamma_{i,m}(\mu) = \beta_i + (1 - \beta_i)\mu_m$$

and $\mathcal{P} : [0, 1]^{JM} \to \Delta_M^J$ such that for $\gamma \in [0, 1]^{JM}$

$$\mathcal{P}_{i,m}(\gamma) = \gamma_{i,m} \prod_{m' : a_{m'} \succ_i a_m} (1 - \gamma_{i,m'})$$

Then, for any $\mu \in \Delta_M$, we can define $p \in \Delta_M^J$ where $p = \Phi(\mu)$, and $\Phi = \mathcal{P} \circ \Gamma$. Then, let $\nu : \Delta_M^J \to \Delta_M$ be the map that averages choice probabilities. That is:

$$\nu_m(p) = \frac{\sum_{i\in N^s} p_{i,m}}{J}$$

[16] This allows us to construct a well-defined mapping from average choice probabilities to choice probabilities to average choice probabilities, which underlies our ability to examine this problem in a smaller dimensional space.

Letting $F = \nu \circ \Phi$, see that μ^* is a fixed point of F if and only if $\Phi(\mu^*)$ is a PIA equilibrium. Since Γ, $\mathcal{P}$, and ν are all continuous mappings, so is $F = \nu \circ \mathcal{P} \circ \Gamma$. Since Δ_M is a compact, convex subset of $\mathbb{R}^M$, F has a fixed point by Brouwer's fixed point theorem. Further, by Lemma A.1, any fixed point lies in $\text{Int}(\Delta_M)$. $\square$

The proof of this result recasts the problem of finding a PIA equilibrium to one of finding average choice probabilities consistent with a PIA equilibrium. We can do so, because the PIA mechanism defines attention probabilities, and therefore choice probabilities, as a function of average choice probabilities in the cluster. This allows us to identify a PIA equilibrium as a fixed point of a self-map on the space of average choice probabilities defined via the PIA mechanism.

3.2 Uniqueness

Now, we look at the question about the uniqueness of PIA equilibrium. Our analysis here provides strong evidence that uniqueness is indeed a structural property of the PIA model. However, to establish this analytically in all its generality is not straightforward. We, therefore, explore this question via varying routes, including numerical methods, highlighting in the process the difficulties in obtaining a single result of the most general form. We first show that PIA equilibria are unique in all menus of two alternatives. Then, we establish uniqueness by identifying conditions under which the mapping F, introduced in the proof of Theorem 3.1 can be shown to be contractive under a norm. We show that these conditions—which essentially amount to bounding the influence-immunity coefficients away from zero—have bite by establishing that there is *no norm* w.r.t. which F is contractive over the entire parameter space.[17] Finally, we provide a degree-theoretic argument for why uniqueness holds under weaker conditions, and provide numerical evidence in their support where analytical routes prove insufficient.

Before starting the discussion, note that when it comes to investigating uniqueness, we only need to consider the case when the most preferred alternative in a menu is not common to all individuals in the cluster. As the proof of Theorem 3.1 establishes, when this is not so, uniqueness is guaranteed. First, analyzing the simplest case, we consider menus of two alternatives. We find that PIA equilibria are unique in such menus under all parameterizations.

Proposition 3.1. *If $A \subseteq X$ is a two alternative menu, a PIA equilibrium in A is unique.*

Proof: Refer to Appendix Section A.3

Generalizing this result to arbitrary menus is most conveniently analyzed through the fixed points of the map F. The most standard route in this form of analysis is to

[17] As the discussion that follows highlights, it is the averaging of choice probabilities over individuals in the cluster that destroys the triangularity and contractive properties of the mapping. Even if we were to analyze this problem through an equilibrium mapping specified in terms of choice probabilities instead of their cluster averages, the same structure would persist.

prove that F is contractive under some metric. Our next result shows that this is indeed true under the supremum norm when immunity from influence coefficients satisfy a lower bound.

Theorem 3.2. *Let $\beta_{\min} = \min_{i \in N^s} \beta_i$. If $\beta_{\min} > \frac{3-\sqrt{5}}{2}$, then F is a contraction with respect to the ℓ^∞ norm. Consequently, a PIA equilibrium in any menu $A \subseteq X$ is unique.*

Proof: Refer to Appendix Section A.4

The difficulty in extending this result to all parameterizations arises from a combination of low values of β_i and the arbitrary form of disagreement in preferences that can occur within each cluster, especially over large menus. While changes in average choice probabilities lead to strictly smaller changes in attention probabilities, changes over multiple alternatives compound, and the non-linearity in (2) can produce large changes in the choice probability of some alternative. A clean demonstration of this is through an analysis of the spectral radius of the Jacobian of F, which we denote by $\rho(D_F)$.

Since $\rho(D_F)$ is the infimum of $||D_F||$ over all operator norms, for F to be a contraction under any norm induced metric, $\rho(D_F) < 1$ is a necessary condition. Define P_i as the mapping which reorders the basis of $\mathbb{R}^M$ in the order of i's preference ranking. Then, the mapping $P_i \Phi_i P_i^{-1}$ is simply the mapping Φ_i reordered such that both the input and output are ordered according to $\succ_i$. By (2), $|D_{P_i \Phi_i P_i^{-1}}|$ is lower triangular and has largest diagonal element $1 - \beta_i$. This implies $\rho(D_{\Phi_i}) = 1 - \beta_i < 1$. However, when averaging across individuals with different preferences, this triangularity breaks. In fact, the following examples show that $\rho(D_F)$ is indeed greater than 1 at certain points on the domain under some parameterizations that do not satisfy the antecedent of Theorem 3.2.[18] These examples are motivated by the fact that when there are exactly two alternatives and two individuals in the cluster, F is indeed a contraction. But moving from two to three, either in the number of alternatives or individuals, no longer guarantees this.

Example 3.1 (Three alternatives)**.** Take $N^s = \{1, 2\}$ with $\beta_1 = 0.2$, $\beta_2 = 0.001$, and let $A = \{a, b, c\}$ with preferences $a \succ_1 c \succ_1 b$ and $b \succ_2 c \succ_2 a$. Evaluate D_F at $\mu = (\mu_a, \mu_b, \mu_c) = (0.93, 0.02, 0.02)$, an interior point of Δ_3 (the default has probability 0.03). A direct computation gives

$$D_F(0.93,\ 0.02,\ 0.02) \ = \begin{pmatrix} 0.87876 & -0.45482 & -0.45482 \\ -0.06774 & 0.51706 & -0.00484 \\ -0.08640 & -0.01048 & 0.51142 \end{pmatrix},$$

whose eigenvalues are approximately 1.01630, 0.49968, and 0.39127. Hence $\rho(D_F(\mu)) \approx 1.01630 > 1$, ruling out contraction in any norm. The mechanism is that agent 2, with

[18]These examples do not imply that F is not a contraction under a metric that is not induced by a norm. Moreover, Proposition 3.1 implies that example 3.2 still admits a unique PIA equilibrium.

$\beta_2 \approx 0$, is effectively choosing deterministically at μ — given μ_a is large and μ_b and μ_c are near-zero, she ends up putting most of her probability mass on a (rather than on her top-ranked b). The resulting off-diagonal column-a entries of D_F are large in magnitude, and combined with the diagonal entries close to one, push the spectral radius above unity.

Example 3.2 (Three individuals)**.** Take $A = \{a, b\}$ with $N^s = \{1, 2, 3\}$, $\beta_1 = 0.2$, $\beta_2 = \beta_3 = 0.001$, and preferences $a \succ_1 b$, $b \succ_2 a$, $b \succ_3 a$. Evaluate D_F at $\mu = (\mu_a, \mu_b) = (0.9, 0.05)$, an interior point of Δ_2 (the default has probability 0.05). A direct computation gives

$$D_F(0.9,\ 0.05) \ = \ \begin{pmatrix} 0.89874 & -0.59947 \\ -0.06400 & 0.68733 \end{pmatrix},$$

with characteristic polynomial $\lambda^2 - 1.58607\,\lambda + 0.57939$ and eigenvalues $\lambda_+ \approx 1.01558$ and $\lambda_- \approx 0.57049$. Hence $\rho(D_F(\mu)) \approx 1.01558 > 1$, again ruling out contraction in any norm. As in Example 1, the failure traces to two near-deterministic agents (with $\beta_i \approx 0$) whose collective influence is not suppressed by the lone moderate-β agent. Holding the preferences fixed and letting $\beta_2, \beta_3 \to 0^+$, a direct computation at the vertex $\mu = (1, 0)$ shows that $\rho(D_F(\mu))$ exceeds one precisely when $\beta_1 < 1/2$, and by continuity an interior neighbourhood of the vertex inherits this.

To extend the uniqueness of the PIA equilibrium to more general cases, we must then explore a route that does not depend on F being contractive or $\rho(D_F) < 1$ on Δ_M. That is what our next result buys us. We use a degree theoretic argument to show that for establishing uniqueness, it is enough for $\rho(D_F) < 1$ to hold at every fixed point, not on the entire simplex.

Proposition 3.2. *Let $A \subseteq X$ be a menu and N^s a cluster such that $\{a : \exists i \text{ s.t. } a = \max_{\succ_i} A\}$ is not a singleton. If at every fixed point μ^* of F,*

$$\rho\left(D_F(\mu^*)\right) \ < \ 1,$$

then F has a unique fixed point in Δ_M, and consequently the PIA equilibrium in A for the cluster N^s is unique.

Proof: Refer to Appendix Section A.5.

When $\beta_{\min} > \frac{3-\sqrt{5}}{2}$, we find as an implication of Theorem 3.2 that $\rho(D_F) < 1$ at all points in Δ_M. However, when this condition is violated, we found examples where, at some points in Δ_M, $\rho(D_F) > 1$. Proposition 3.2 provides another route for proving the uniqueness of PIA equilibria by relaxing the requirements on $\rho(D_F)$. While we do not have an analytical proof establishing that $\rho(D_F(\mu^*)) < 1$ at every fixed point μ^* under an arbitrary parameterization, Appendix A.6 provides thorough numerical evidence in favor of this.

4 Identification

We now address the question about the identification of the parameters of the PIA model. This exercise corresponds to an important concern in the literature about the identification of structural parameters underlying peer effects. A less addressed but equally relevant question is about the identification of the set of peers itself. There is a large econometrics literature addressing the first question and an emerging one looking at the second. The approach to identification we pursue here is based on decision-theoretic tools that rely on behavioral variations across different menus of alternatives. That is, consider a revealed-preference/experimental design set-up that generates data on a profile of choice probabilities across menus. Suppose an analyst maintains that the dataset is consistent with the PIA model. The question of identification is one about whether the analyst can draw on the inter-menu variations in the data to uniquely elicit the model's parameters. We show that for the PIA model this is indeed the case, provided the dataset is rich enough in a precise sense that we outline below. The novel insight that emerges from this analysis is regarding how the structure of socially generated attention and homophily interact to pave the path to identification.

To set up the identification question formally, let $\mathcal{X}$ be a collection of non-empty subsets of X (i.e., menus of alternatives) over which the analyst has data on the profile of choice probabilities. A dataset is a joint random choice rule $p : \mathcal{X} \to \cup_{A \in \mathcal{X}}[0, 1]^{|A| \times n}$, where for any $A \in \mathcal{X}$,

$$p(A) = \left\{ (p_i(a,\, A))_{a \in A} \,|\, \sum_{a \in A} p_i(a, A) \leq 1, i \in I \right\}$$

Let Π be the collection of all such joint random choice rules. Let Ξ be the set of all possible collections of model parameters. That is, $\xi \in \Xi$ is a tuple of preference rankings, a homophilous clustering, and immunity from influence coefficients,

$$\xi = \left(\{\succ_i : i \in I\}, N = \left\{ N^1, \ldots, N^S \right\}, \{\beta_i : i \in I\} \right)$$

Definition 4.1. *Let $\xi, \xi' \in \Xi$ and $p \in \Pi$ be a PIA equilibrium corresponding to both ξ and ξ'. The parameters of the PIA model are uniquely identified if for any such ξ and ξ', $\xi = \xi'$.*

In other words, the question that identification addresses is whether a dataset consisting of an equilibrium choice profile of the PIA model maps back to a unique set of PIA parameters or are multiple such parametrizations possible? The model is well-identified if the parameterization is unique. The following result shows that this is indeed the case.

Theorem 4.1. *If $\mathcal{X}$ contains all two alternative menus, then the parameters of the PIA model are uniquely identified.*

Proof: Please refer to Appendix A.7.

We show in the proof that the identification problem can be broken down into three subproblems. First, the identification of the clusters; second, the identification of individual preferences conditional on the identification of the clusters; and third, the identification of the immunity from influence coefficients, conditional on the identification of clusters and preferences. Working backwards, let's look at the third problem first. Suppose the analyst has succeeded in identifying the clustering $(N(i))_{i \in I}$, and individual preferences, $(\succ_i)_{i \in I}$. She can then determine for any $i \in I$ and any menu A, the average probability with which any alternative $a \in A$ is chosen in i's cluster. Denote this by $\mu_{N(i)}(a, A) = \frac{1}{|N(i)|} \sum_{i \in N(i)} p_i(a, A)$. (In the way of notation, note that for any $J \subseteq I$, we will let $\mu_J(a, A) := \frac{1}{|J|} \sum_{i \in J} p_i(a, A)$ denote the average probability of alternative a being chosen in menu A among individuals in J). Clearly, as long as the clusters are uniquely identified, these average choice probabilities are as well. Now, consider any menu A in which i chooses stochastically, i.e., $p_i(b, A) \neq 1$, for any $b \in A$, and let $a = \max_{\succ_i} A$. Then, it can be shown that i's susceptibility to influence, $1 - \beta_i$ is given by,

$$1 - \beta_i = \frac{1 - p_i(a, A)}{1 - \mu_{N(i)}(a, A)}$$

The numerator, $1 - p_i(a, A)$, gives the probability of i not choosing a in A despite it being her most preferred alternative. The denominator specifies the average probability of a not being chosen in A in her cluster. Therefore, the ratio gives a measure of the extent to which a not being chosen in her cluster translates to i not choosing it, despite it being her most preferred alternative in A. Therefore, it reveals the extent of her susceptibility to influence. Accordingly, as long as an individual's cluster and her preferences can be uniquely identified by the analyst, she can also uniquely identify her immunity from influence coefficient β_i.

Now, moving to the second problem of identifying an individual's preference, given the knowledge of the individual's cluster, we show in Lemma A.2 and Corollary A.2 and A.3 that, for any $i \in I$ and $a, b \in X$,

$$a \succ_i b \iff \frac{1 - p_i(a, ab)}{1 - p_i(b, ab)} \leq \frac{1 - \mu_{N(i)}(a, ab)}{1 - \mu_{N(i)}(b, ab)}$$

In other words, the notion of revealed preference in the model is the following. For any individual i, and any pair of alternatives, $a, b \in X$, if the ratio of the probability of i not choosing a to not choosing b in the menu $\{a, b\}$ is no greater than the corresponding ratio for her cluster, then i prefers a to b. Accordingly, as soon as the analyst is able to identify any individual's cluster, she is able to identify her preferences as well.

Therefore, the problem of uniquely identifying the parameters of the model boils down to the analyst's ability to identify the clusters. What makes such identification possible?

There are two features of the model that are key to this identification. The first is a property of the attention function pertaining to an alternative receiving attention with certainty. Observe that under our attention function, as the average probability of an alternative being chosen in a cluster goes to one, the probability with which this alternative receives attention also goes to one. This property of the attention function allows for the possibility that when everyone within a cluster shares the same top-alternative in a menu then, *in equilibrium*, they end up paying attention to this alternative for sure and, accordingly, choosing it for sure, with both tendencies reinforcing each other. The second feature of the model that is crucial for identification is preference homophily in the clustering. Homophily ensures that between any two clusters there exists alternatives, over which everyone's preferences in one cluster agree but at least someone in the other cluster disagrees with these preferences. In combination with the first feature pertaining to attention with certainty, homophily then produces a pattern of deterministic choices in an otherwise stochastic choice environment that can be used to identify network links. To see how these two features of the model work hand in hand and, in particular, what may go wrong as regards identification when the clusterings are not homophilous, consider the following example.

Example 4.1. Suppose $X = \{a, b, c\}$ is the set of alternatives and $I = \{1, 2, 3, 4\}$ the set of individuals in society who are partitioned into the clustering $N = \{\{1, 3\}, \{2, 4\}\}$. Further let their preferences be given by: $a \succ_1 b \succ_1 c$, $a \succ_2 b \succ_2 c$, $a \succ_3 c \succ_3 b$, $a \succ_4 c \succ_4 b$; and their immunity from influence coefficients by $\beta_1 = \beta_2 = 3/5$ and $\beta_3 = \beta_4 = 2/5$. Observe that the clustering is not homophilous, as a is the common best alternative for all individuals and neither cluster agrees on the ranking $\{b, c\}$. Applying the structure of interactions underlying a PIA equilibrium (equations 1 and 2 underlying Definition 2.1) gives us the following profile: $p_i(a, A) = 1$ for all $i \in I$ and A such that $a \in A$, and for the menu $\{b, c\}$, choice probabilities are given by,

	p_1	p_2	p_3	p_4
b	$\frac{\sqrt{1801}-31}{14}$	$\frac{\sqrt{1801}-31}{14}$	$\frac{2\sqrt{1801}-83}{7}$	$\frac{2\sqrt{1801}-83}{7}$
c	$\frac{7\sqrt{1801}-283}{102}$	$\frac{7\sqrt{1801}-283}{102}$	$\frac{\sqrt{1801}-21}{34}$	$\frac{\sqrt{1801}-21}{34}$

However, since individuals 1 and 2 are replicas of one another, as are 3 and 4, the clustering given by $\hat{N} = \{\{1, 4\}, \{2, 3\}\}$, along with the same preferences and influence coefficients, also rationalizes the same profile of choice probabilities through the PIA mechanism.

What is it that frustrates the analyst's attempt to exactly identify the clusters in this example? To answer this question, note a particular pattern of stochastic and deterministic behavior among individuals in society that the model implies. Since all individuals have a well-defined preference ranking over the set of alternatives, they would in the absence of peer influence deterministically choose the best alternative in any menu according to their preferences. Hence, the presence of stochastic behavior suggests the possibility of peer influence through the random attention channel. At the same time, it is possible that even with peer influence they choose deterministically. However, for them to do so under a PIA equilibrium, as highlighted above by the first key feature of the model underlying identification, what needs to be true is that everyone else in their cluster does likewise (Corollary A.1). In other words, the analyst may get an "upper bound" of an individual's cluster by identifying all those who choose an alternative from a menu with probability one whenever she does so. But to get exact identification, say, differentiate cluster N^s from N^t, there needs to be some menu where only those in N^s choose deterministically, but those in N^t don't. It is precisely here that homophilous clusterings come to the analyst's aid in identification. Homophilous clusterings, by ensuring that between any two clusters there exists at least some alternatives over which one cluster fully agrees on preferences but that agreement is not shared in the other, guarantees that exact identification of clusters is possible. Going back to the example, we see this clearly as the menus in which individuals choose deterministically are the same for all of them. These are precisely those menus that include alternative a, which is chosen with probability one in all of them. Based on this choice data, the best that the analyst can do is to over-identify everyone's cluster as all of society. However, if the underlying clustering is not homophilous, she can't improve upon this upper bound any further.

To the best of our knowledge, this insight about the possibility of network identification by drawing on socially generated attention and preference homophily is new and we think future work can tap into this route for identification. Of course, in a full-fledged empirical analysis with additional exogenous shocks, the sharp divide between deterministic and stochastic choices that we get here may not hold. Even then, the broader strategy of identification by analyzing the structure of endogenous randomness in behavior brought about by socially generated attention in an environment with stable preferences should carry over. In this regard, homophily by modulating the structure of this randomness differentially within and across clusters can be the critical key to identification.

We end this section with two final comments. First, it should be noted that even without the homophily assumption, there are profiles of choice probabilities that are rationalized by a unique collection of PIA model parameters. In Example A.1 of Appendix A.9, we illustrate this point. However, so long as there are at least 4 individuals in society, Example 4.1 highlights the manner in which we can construct a profile of choice probabilities that can be represented by multiple clusterings. Second, our result also highlights that an analyst need not obtain choice data on all menus to be able to identify the parameters of the PIA model. Having data on all binary menus suffices for unique identification. Example A.2 of Appendix A.9 shows why binary menus are indispensable for identification.

5 Robustness

In the analysis thus far we have assumed a particular linear formulation of the attention function, $\gamma_i(a, A) = \beta_i + (1 - \beta_i)\mu_{N(i)}(a, A)$. The reader may have wondered about how crucial is this linear formulation to the findings of the last two sections. In this section we address this robustness question, first in relation to existence and uniqueness of PIA equilibrium, and then regarding identification. Of course, once we get outside the linear formulation, there is no telling what exact structure the non-linear relationship between popularity and attention should take. Even so, we think that a reasonable non-linear generalization of this functional form would be to some $f_i : [0, 1] \to [0, 1]$ that satisfies (i) $f_i(\mu_{N(i)})$ is strictly increasing and continuous, (ii) there is some baseline attention $f_i(0) > 0$, and (iii) $f_i(\mu_{N(i)}) > \mu_{N(i)}$ for $\mu_{N(i)} < 1$, i.e., the probability of paying attention to an alternative is larger than the cluster's average probability of choosing it. Observe that the linear attention function satisfies these properties.

5.1 Existence and uniqueness

As far as existence goes, the obvious thing to note is that the continuity of f_i is enough to guarantee the existence of a PIA equilibrium. On the other hand, the nonlinearities in $(f_i)_{i\in I}$ generate multiplicities in PIA equilibria. The linearity of the attention function, with its slope $1 - \beta_i < 1$, played an important role in establishing uniqueness. Without that multiplicities persist.

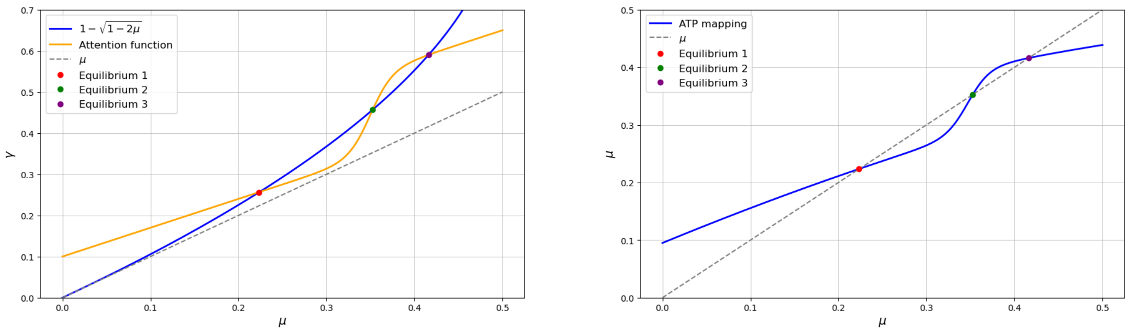
To illustrate this, we consider the simple case with two alternatives, two individuals with opposing preferences, and a common attention function. To make our point, we focus on symmetric equilibrium under which the average probability of choosing each alternative is the same $\mu \in \left[0, \frac{1}{2}\right]$. It can be shown that under any such equilibrium, the attention probability on each alternative is $\gamma = 1 - \sqrt{1 - 2\mu}$.[19] On the other hand, the attention function implies $\gamma = f(\mu)$. Then, a symmetric PIA equilibrium is a root of $f(\mu) - 1 + \sqrt{1 - 2\mu}$. By $f(0) > 0$ and $f(1/2) < 1$, we know that such a symmetric PIA equilibrium exists. For S-shaped attention functions, we can find multiple symmetric equilibria. Figure 1a highlights one such example.

The key feature of such an attention function is a steep rise in attention probabilities after a threshold, with flatter regions on either side. This threshold can be interpreted as a *virality threshold*, beyond which the alternative rapidly gains attention. Since we know that $f(0)$ and $f(1/2) - 1$ have different signs, there must be (except for tangencies) an odd number of equilibria. The first, low attention equilibrium is found in the initial flat region. Then, the steep rise in attention creates a second equilibrium, with the subsequent flattening of the function producing a third equilibrium. Figure 1b shows

[19] The probability of choosing the preferred alternative is γ, and the other alternative is $\gamma(1-\gamma)$. Then $2\mu = \gamma(2 - \gamma)$, which implies $\gamma = 1 \pm \sqrt{1 - 2\mu}$. Since $\gamma \in [0, 1]$, we only consider the smaller root.

how the attention function maps average probabilities to new average probabilities. In doing so, using the slope of the mapping, we know that the first and third equilibria, on the flat regions of the attention function, are stable, while the second equilibrium, in the steep region, is unstable.

Figure 1: S-shaped attention function



(a) Multiple equilibria

(b) Stability of equilibria

5.2 Identification

Next, we consider the question of robustness of the identification exercise to generalizations of the attention function. Is the result establishing the exact identification of the clusters and preferences still true under the generalized attention function introduced above? For the clustering, the answer is yes. But, when it comes to preferences, we may only get partial identification. Recall that in the linear case, along with homophily, the exact identification of the clusters relied on the particular property of the attention process under which, as the average probability of an alternative being chosen in one's cluster goes to one, the probability of that alternative receiving her attention also moves towards certainty. The generalized attention function retains this property. Therefore, this property, along with homophily, once again guarantees the exact identification of the clusters.

On the other hand, the strategy that we adopted above to identify preferences crucially depends on the linear parametric form of the attention function. To appreciate this point, assume we observe a dataset of choice probabilities that are generated by the PIA model with generalized attention functions.[20] Suppose clusters have been exactly identified, following the argument made above. Since we observe choice probabilities, we can compute average choice probabilities for each alternative, in any menu, in each cluster. Under the linear specification, we use the equivalence $a \succ_i b \iff \frac{1-p_i(a,ab)}{1-p_i(b,ab)} \leq \frac{1-\mu_{N(i)}(a,ab)}{1-\mu_{N(i)}(b,ab)}$ to identify preferences, but this no longer works with generalized attention

[20] Note that such a dataset would feature some properties that are also characteristic of PIA equilibrium datasets with linear attention functions. Particularly, $p_i(a, A) > 0$ for all $a \in A$ and $\sum_{a \in A} p_i(a, A) < 1$ if there exists some $a' \in A$ such that $p_j(a', A) \in (0, 1)$ for some $j \in N(i)$.

functions. If the generalized attention functions were known, we could use the fact that $f_i(\mu_{N(i)}(a,\, A)) = \frac{p_i(a,\, A)}{1-\sum_{a' \succ_i a : a' \in A} p_i(a',\, A)}$ under a PIA equilibrium to uniquely identify i's preferences.[21] However, for an arbitrary parametric family of attention functions, jointly identifying preferences and attention parameters uniquely is not guaranteed. This problem is only made worse when considering a general, non-parametric class of functions. That said, the theoretical properties of f_i impose rigid bounds that allow for partial identification of preferences, and under certain data variations, exact identification. In particular, we can propose the following procedure for preference elicitation through elimination.

For any individual i, start with the hypothesis that this individual's preference is some $\succ_i$. Then derive what the implied attention probability for any alternative, in any menu is, given the hypothesized preference relation and the data on choice probabilities. Finally, check whether these implied attention probabilities are consistent with the constraints imposed by the theoretical properties of the generalized attention function. Specifically, for any assumed preference $\succ_i$ and choice data p_i, individual i's implied attention probability for alternative a in menu A is

$$\hat{\gamma}_i(a,\, A) = \frac{p_i(a,\, A)}{1 - \sum_{a' \succ_i a} p_i(a',\, A)} > 0$$

if $\sum_{a' \succ_i a} p_i(a',\, A) \neq 1$. The analyst knows that for choice data consistent with a PIA equilibrium, there exists a generalized attention function f_i that satisfies conditions (i)-(iii) such that $f_i(\mu_{N(i)}(a,\, A)) = \hat{\gamma}_i(a,\, A)$ for all $A \in \mathcal{X}$ and $a \in A$. The analyst may not know the exact form of this attention function but knows the properties it satisfies, which allows her to impose certain constraints on the relationship between these implied attention probabilities and the average choice probabilities seen from the data in the cluster. Specifically,

1. $\hat{\gamma}_i(a,\, A) \underset{(\leq)}{\geq} \hat{\gamma}_i(a',\, A') \iff \mu_{N(i)}(a,\, A) \underset{(\leq)}{\geq} \mu_{N(i)}(a',\, A')$ for all $A, A' \in \mathcal{X}$, $a \in A$, $a' \in A'$

2. $\hat{\gamma}_i(a,\, A) \geq \mu_{N(i)}(a,\, A)$ for all $A \in \mathcal{X}$, $a \in A$

Statement 1 says that for any two alternatives, potentially in different menus, their implied attention probabilities must be monotonic with respect to average choice probabilities. Statement 2 puts a lower bound on implied attention probabilities, which must always be weakly larger than average choice probabilities.

[21] By the restrictions on the attention function, $p_i(a,\, A) = f_i(\mu_{N(i)}(a,\, A))$ if and only if a is i's most preferred alternative. Then, note that the probability of choosing an alternative conditional on not choosing the best $k-1$ alternatives is its attention probability if and only if it is the k-th best alternative. Apply this relation recursively to recover preferences entirely.

Let $\mathcal{P}_i$ be the set of all strict rankings such that i's implied attention probabilities and average choice probabilities in the cluster satisfy conditions (1) and (2) above. i's preferences are uniquely identified if and only if $\mathcal{P}_i$ is a singleton. The following examples provide an illustration of how preferences may or may not be uniquely identified, given a PIA equilibrium.

Example 5.1 (Exact Identification)**.** Let $X = \{a, b\}$, $\mu_{N(i)}(a, X) = 0.2$, $\mu_{N(i)}(b, X) = 0.6$, with $p_i(a, X) = 0.3$ and $p_i(b, X) = 0.49$. If $a \succ_i b$, it must be that $\hat{\gamma}_i(a, X) = 0.3$ and $\hat{\gamma}_i(b, X) = 0.49/0.7 = 0.7$. If $b \succ_i a$, $\hat{\gamma}_i(b, X) = 0.49 < \mu_{N(i)}(b, X)$! Hence, $\succ_i$ is exactly identified as $a \succ_i b$.

Example 5.2 (No identification)**.** Consider the same setup as before, but now with $\mu_{N(i)}(a, X) = 0.15$, $\mu_{N(i)}(b, X) = 0.25$, and $p_i(a, X) = 0.2$, $p_i(b, X) = 0.4$. If $a \succ_i b$, then $\hat{\gamma}_i(a, X) = 0.2$ and $\hat{\gamma}_i(b, X) = 0.5$. If $b \succ_i a$, then $\hat{\gamma}_i(b, X) = 0.4$ and $\hat{\gamma}_i(a, X) = 1/3$. In both cases, $\hat{\gamma}_i(b, X) > \hat{\gamma}_i(a, X)$ and $\hat{\gamma}_i \geq \mu_{N(i)}$. Then, both preferences rationalize choice probabilities (with an appropriately chosen attention function) as a PIA equilibrium.

Finally, note that, in the absence of parametric restrictions on f_i, for any $\succ_i \in \mathcal{P}_i$, we can construct an arbitrary increasing, continuous attention function that lies above the 45 degree line, with $f_i(0) > 0$, such that f_i and $\succ_i$ rationalize i's choice probabilities as being generated under a PIA equilibrium. So identifying the exact form of f_i from the data is not feasible.

6 Falsifiability

We now establish that our model is falsifiable based on observable choice data. Such an exercise provides the analyst with the tools to determine whether choice data suggestive of peer effects can be rationalized by the PIA mechanism or not and, if it can be, serves as an empirical justification for its specific details such as a linear attention function and the exact form of preference–attention interaction. To that end, we provide conditions on this data that behaviorally characterize the model. In other words, suppose an analyst has a dataset comprising a profile of choice probabilities over some set of menus $\mathcal{X}$, captured by a joint random choice rule $p : \mathcal{X} \to \cup_{A \in \mathcal{X}} [0, 1]^{|A| \times n}$. What conditions must the dataset p satisfy for the analyst to maintain the hypothesis that it was generated by the interactions underlying the PIA model? Equivalently, when can this analyst reject this hypothesis? We provide three conditions on this dataset that fully answer these questions.

The first key idea underlying this exercise is the following. In our set-up, each individual has a single set of well-defined preferences; hence, without any other consideration or influence, she should choose deterministically her most preferred alternatively in any menu. Therefore, her choosing stochastically reveals that others may influence her behavior, given the random attention that such influence generates. In other words, the constraint imposed by influence on behavior is that it prevents individuals from *independently* choosing their most preferred alternative for sure from a menu like a standard rational decision maker, and stochastic choice serves as a signature of this influence. To understand this better, suppose a is i's most preferred alternative in a menu, but her peer j has a positive probability of choosing b in that menu. This being so, j could potentially draw i's attention away from a and towards b, thereby introducing a *chance* of i choosing this alternative and ensuring that her probability of choosing a is no longer one. Viewed from the opposite direction, the only scenario, then, under which i may end up choosing a for sure is if j doesn't draw her attention towards other alternatives through her behavior, i.e., j too chooses a for sure. This reasoning, therefore, suggests a simple way of behaviorally determining who is connected to whom in the influence network. Any $i, j \in I$ are (revealed to be) *connected* if for any $A \in \mathcal{X}$ and $a \in A$, $p_i(a,\, A) = 1$ if and only if $p_j(a,\, A) = 1$.[22] Our first restriction on the dataset p brings together these interconnected inferences revealed through the co-existence of stochastic and deterministic choices—stochasticity in behavior reveals that individuals are influenced, and deterministic behavior within it reveals the connections underlying that influence. Note that we say that $i \in I$ chooses stochastically if there exists some $A \in \mathcal{X}$, s.t. $p_i(a, A) \neq 1$, for any $a \in A$.

Axiom 1 (**Peer influence**). *Each $i \in I$ chooses stochastically, and for any such i there exists $j \neq i$ such that i and j are connected.*

Denote the set of individuals to whom $i \in I$ is connected by $R(i)$. That is,

$$R(i) = \{j \in I : p_i(a,\, A) = 1 \iff p_j(a,\, A) = 1, A \in \mathcal{X},\, a \in A\}$$

Further, define for any $A \in \mathcal{X}$ and $a \in A$, the average choice probability of a being chosen in A in $R(i)$ by:

$$\mu_{R(i)}(a, A) = \frac{1}{|R(i)|} \sum_{j \in R(i)} p_j(a, A)$$

Next, we ask under what restrictions does an individual's behavior reveal that she has consistent preferences. Suppose, for some menu A and $a \in A$, $p_i(a, A) \geq p_i(b, A)$ or, equivalently, $\frac{1-p_i(a,A)}{1-p_i(b,A)} \leq 1$, for all $b \in B \setminus a$. From this, can we infer that a is i's most preferred alternative in A? Well, not necessarily, because the choice probabilities, $p_i(a, A)$ and $p_i(b, A)$, may reflect not just i's preference but also the relative popularity of a vs. b amongst i's connections, as captured by $\mu_{R(i)}(a, A)$ and $\mu_{R(i)}(b, A)$, respectively. Hence, to make a correct inference, one needs to control for this. One may do so by comparing the probability ratio of i not choosing a to not choosing b, $\frac{1-p_i(a,A)}{1-p_i(b,A)}$, to the corresponding ratio

[22] In general, this would be a necessary condition for a connection, but in a set-up with homophily it is also sufficient.

capturing average choice behavior among her connections, $\frac{1-\mu_{R(i)}(a,A)}{1-\mu_{R(i)}(b,A)}$. In particular, if the first ratio is no larger than the second for any $b \in A \setminus a$, then it reveals that a is indeed i's most preferred alternative in A. Our second restriction on the choice data draws on the standard independence of irrelevant alternatives (IIA) condition for deterministic choices to introduce the requirement that such inferences about an individual's preferences should not be menu dependent.

Axiom 2 (Stochastic IIA). *For all $i \in I$ and $A \in \mathcal{X}$, $\exists!\ a \in A$ such that $\frac{1-p_i(a,A)}{1-p_i(b,A)} \leq \frac{1-\mu_{R(i)}(a,A)}{1-\mu_{R(i)}(b,A)}, \forall b \in A \setminus a$; and for any $B \subseteq A$, $B \in \mathcal{X}$, with $a \in B$, $\frac{1-p_i(a,B)}{1-p_i(b,B)} \leq \frac{1-\mu_{R(i)}(a,B)}{1-\mu_{R(i)}(b,B)}, \forall b \in B \setminus a$.*

That is, in any menu A, there exists a unique alternative $a \in A$ that is revealed to be the most preferred, and in any sub-menu B of A in which a is present, the same is true. Observe that, among other things, the axiom rules out the possibility that for any $b \neq a, b \in A$, $p_i(b,A) = 1$ and, correspondingly, the possibility that $\mu_{R(i)}(b,A) = 1$, as these would make the inequality undefined.[23] This restriction, of course, is intuitive—if $a \in A$ is revealed to be i's most preferred alternative, then such a revelation should, at the very least, rule out the possibility that she chooses some other alternative in A with probability one.

Our final restriction on the choice data imposes the requirement that an individual's susceptibility to influence should be independent of the alternative under consideration or the menu in which this alternative appears. To understand how such a requirement can be imposed through a choice-based restriction, note that for any menu A in which i chooses stochastically, if $a_1 \in A$ is i's most preferred alternative in the menu, then the ratio $\frac{1-p_i(a_1,A)}{1-\mu_{R(i)}(a_1,A)}$ captures the extent of i's susceptibility to influence when it comes to her likelihood of choosing a_1. For instance, suppose $p_i(a_1,A) = 0.9$ and $\mu_{R(i)}(a_1,A) = 0.6$. Then it means that a 40% chance of a_1 not being chosen on average amongst her connections translates into a 10% chance of this alternative not being chosen by i, even though it is her most preferred alternative. Hence, the ratio $\frac{0.1}{0.4} = \frac{1}{4}$ captures i's idiosyncratic susceptibility to influence w.r.t. a_1. Now consider i's second most preferred alternative in A, call it a_2. Note that w.r.t. a_2, $\frac{1-p_i(a_2,A)}{1-\mu_{R(i)}(a_2,A)}$ is not the correct measure of idiosyncratic susceptibility to influence. This is because a_2 is not i's most preferred alternative in A and that is part of the reason why this alternative may not be chosen. Therefore, the correct probability to look at is not the unconditional probability of a_2 not being chosen, $1 - p_i(a_2,A)$, but rather the probability that a_2 is not chosen conditional on no alternative preferred to it, i.e., a_1, being chosen; specifically, the probability $1 - \frac{p_i(a_2,A)}{1-p_i(a_1,A)}$. The susceptibility to influence w.r.t. the choice of a_2 is then captured by the ratio $\frac{1-\frac{p_i(a_2,A)}{1-p_i(a_1,A)}}{1-\mu_{R(i)}(a_2,A)}$. If this susceptibility to influence is independent of the alternative

[23] Likewise, for any $b \neq a, b \in B$.

under consideration, then it should be the case that $\frac{1-p_i(a_1,A)}{1-\mu_{R(i)}(a_1,A)} = \frac{1-\frac{p_i(a_2,A)}{1-p_i(a_1,A)}}{1-\mu_{R(i)}(a_2,A)}$. Our final restriction requires that this idiosyncratic susceptibility to influence should not vary across alternatives in a menu, nor should it vary across menus. To capture this formally, we need to introduce some notation. First, for any menu A in which i chooses stochastically and $a \in A$, define:

$$\bar{A}_i(a) = \left\{ b \in A \setminus a : \frac{1 - p_i(b, ab)}{1 - p_i(a, ab)} \leq \frac{1 - \mu_{R(i)}(b, ab)}{1 - \mu_{R(i)}(a, ab)} \right\}$$

Based on our discussion above, $\bar{A}_i(a)$ contains those alternatives in A that are revealed to be preferred by i to a. Then the probability of a being chosen conditional on no alternative that is revealed to be preferred to it being chosen is given by:

$$p_i^*(a, A) = \frac{p_i(a, A)}{1 - \sum_{b \in \bar{A}_i(a)} p_i(b, A)}$$

Axiom 3 (**Menu independence of influence**). *For any $i \in I$, menus $A, B \in \mathcal{X}$ in which i chooses stochastically, and $a \in A$, $b \in B$,*

$$\frac{1 - p_i^*(a, A)}{1 - \mu_{R(i)}(a, A)} = \frac{1 - p_i^*(b, B)}{1 - \mu_{R(i)}(b, B)} < 1$$

These three conditions together characterize the PIA model. We say that the dataset $p : \mathcal{X} \to \cup_{A \in \mathcal{X}} [0,1]^{|A| \times n}$ is PIA rationalizable if there exists model parameters $(\{\succ_i : i \in I\}, N = \{N^1, ..., N^S\}, \{\beta_i : i \in I\})$ such that for any menu $A \in \mathcal{X}$, $(p_i(., A))_{i \in I}$ is a PIA equilibrium for those parameters.

Theorem 6.1. *Suppose all two and three alternative menus are in $\mathcal{X}$. Then the dataset $p : \mathcal{X} \to \cup_{A \in \mathcal{X}} [0,1]^{|A| \times n}$ is* PIA rationalizable *if and only if it satisfies peer influence, stochastic IIA, and menu independence of influence.*

Proof: Please refer to Appendix A.8.

Remark 6.1. Manzini and Mariotti (2014) note that in an individual choice setting, a random consideration set model with menu-dependent attention probabilities can neither be falsified, nor can the individual's preferences or attention probabilities be identified. In our model, even though attention probabilities are menu-dependent, due to their endogenous determination through peer behavior, the model is indeed falsifiable, as the result above shows. Moreover, the mechanism underlying attention probabilities allows us to uniquely identify an individual's set of peers, and thereby their preferences and immunity from influence coefficients. We leave it to future research to investigate general conditions under which models with menu-dependent attention probabilities are falsifiable.

7 Empirical Implications

We now study some important lessons that our model has for the empirical measurement of peer effects. Our model doesn't consider contextual or correlated effects and, therefore, is not adequate for a full-blown empirical analysis. However, it comprehensively studies endogenous effects and their implications. These implications, especially those generated by the model's non-linearities, will naturally carry over into any fully specified empirical model and would, therefore, need to be accounted for. The leading models through which endogenous peer effects have been measured in the literature are the linear-in-means (LIM) model and the discrete choice model. Under the LIM model, endogenous social effects are captured in terms of a stochastic linear relationship between an individual's outcome or action and the average of the same in her peer group. Discrete choice models capture this through a non-linear specification that makes an individual's probability of choosing an outcome depend on the choice probabilities of her peers. The implications for measurement of peer effects emerging from our model connects to both set-ups. It naturally relates to discrete choice models because our model specifies a non-linear equilibrium relationship between an individual's probability of making a particular choice and that of her peers. Interestingly, it relates to the LIM model as well because when the model is written out in terms of expectations, it implies a quasi-linear relationship between an individual's expected outcomes and that of her peers, with the non-linear part of the relationship a consequence of the crowding out effect discussed earlier. Interactions in our model also generate a *social multiplier*, whose underlying drivers has policy implications.

7.1 A PIA model of test taking: Accounting for non-linearities

To make these observations, we build on the model of effort choice in Section 2. We consider that students taking tests must decide the amount of effort they exert in studying, and that their actions are determined through the PIA mechanism as described there. As in that section, they choose between efforts e_H, e_L, and e^*, and have influence parameters $(\beta_i)_{i \in N^s}$. In order to develop a more comprehensive statistical model accommodating the insights of the PIA model, we introduce a source of exogenous variation by assuming that students are subject to independent taste shocks at the start of every period. Specifically, at the start of the period, each student independently draws a preference for high effort ($e_H \succ_i e_L$) with probability ϱ and preference for low effort ($e_L \succ_i e_H$) with probability $1 - \varrho$.

For behavior determined by the PIA mechanism, students' expected effort levels can be expressed as a function of the average expected effort level of the cohort, their influence parameter, and the preference shocks they receive. In particular, in a given period:

$$\mathbb{E}\left[e_i\right] = \beta_i(e_H + (1-\beta_i)e_L) + (1-\beta_i)\,\mathbb{E}_\mu\left[e\right] - \delta_i \tag{3}$$

for δ_i defined as

$$\begin{aligned}\delta_i &= (1-\beta_i)e_L[\beta_i(\mu_H+\mu_L) + (1-\beta_i)\mu_H\mu_L] \\ &\quad + (e_H - e_L)(\beta_i^2 + (1-\beta_i)\left[\beta_i(\mu_H+\mu_L) + (1-\beta_i)\mu_H\mu_L\right])\mathbb{1}\left[e_L \succ_i e_H\right]\end{aligned}$$

where $\mathbb{E}\left[e_i\right]$ is the expected effort level of student i in the period, and $\mathbb{E}_\mu\left[e\right]$ is the average expected effort level of the cohort.[24] A notable feature of this relationship is that while influence works nonlinearly in our model, the expected effort level of an individual is indeed linear in the expected effort level of her cohort. Moreover, the coefficient $(1-\beta_i)$ is exactly the extent to which the individual's attention probabilities depend on the average choice probabilities in their cluster. As such, it is natural to compare how linear estimation techniques fare against non-linear estimators of the relationship between $\mathbb{E}\left[e_i\right]$ and $\mathbb{E}_\mu\left[e\right]$, and the β_i coefficients. It is straightforward to establish that a maximum likelihood estimator would provide consistent estimates of the model's parameters. On the other hand, an analyst who is unaware of the true data generating process, or wishes to guard against the risk of misspecification, might use a linear-in-means (LIM) model to measure peer effects.

An LIM model of peer effects posits a linear relationship between an individual's expected effort and the average expected effort of her peers:

$$\mathbb{E}\left[e_i\right] = \alpha_i + \lambda_i\,\mathbb{E}_\mu\left[e\right] + u_i$$

Here, α_i is some constant, λ_i is the peer effects coefficient, and u_i is an error term. If the underlying data is generated by the PIA mechanism, this equation takes the form (3), with $\alpha_i = \beta_i(e_H + (1-\beta_i)e_L) - \mathbb{E}\left[\delta_i\right]$,[25] $\lambda_i = 1-\beta_i$, and $u_i = \mathbb{E}\left[\delta_i\right] - \delta_i$. For the peer effects coefficient to be identified using a regression, we require that $\mathbb{E}\left[\mathbb{E}_\mu\left[e\right]u_i\right] = 0$, i.e. the average expected effort and the error term be uncorrelated. However, observe that if the PIA mechanism underlies the true model of behavior, the error u_i contains μ_H, μ_L and $\mathbb{1}\left[e_L \succ_i e_H\right]$, all of which affect the average expected effort in the cohort. As a result, the population regression coefficient on $\mathbb{E}_\mu\left[e\right]$, λ_i^*,[26] from regressing $\mathbb{E}\left[e_i\right]$ on $\mathbb{E}_\mu\left[e\right]$ and a

[24] While we omit the details for notational brevity, the expectations are conditional expectations, conditioned on the realization of a preference profile. That is, $\mathbb{E}\left[e_i\right]$ is truly $\mathbb{E}\left[e_i \middle| (\succ_j)_{j\in N^s}\right]$, which is a random variable as preferences are random. As a result, the relationship expressed above is one between random variables.

[25] This normalization, adjustment by $\mathbb{E}\left[\delta_i\right]$, is not necessary at this stage, but makes the error, u_i, mean zero and thus an intuitive object.

[26] Defined as $\lambda_i^* \equiv \frac{Cov(\mathbb{E}[e_i],\,\mathbb{E}_\mu[e])}{Var(\mathbb{E}_\mu[e])}$

constant, is not equal to λ_i. By extension, the OLS estimator of λ_i, whose probability limit is λ_i^*, is not consistent for λ_i, or $1-\beta_i$.

In fact, we can say more about the nature of this estimator. Note that the LIM estimator can be operationalized in two ways, with the regressor including individual i in the average, or excluding i.[27] The latter may be preferred to avoid an additional channel of simultaneity in the regression equation.[28] We run a Monte Carlo simulation with both forms of the LIM estimator, where we generate data through the PIA model and measure the peer effects coefficient using the LIM estimator, and find that for $\varrho \geq 1/2$, both overestimate the peer effect coefficient on average.[29] Predictably, the estimator that includes i in the average produces an even larger positive bias. In Appendix A.10, under the added assumption that β is common across individuals, we show analytically that the asymptotic bias of the LIM estimator is positive if and only if the ratio of the variance of average expected effort and expected variance of individuals' expected efforts conditional on the preference distribution is large enough (these quantities, in turn, depend on ϱ). The threshold, in particular, is decreasing in β and cluster size.

To see how features of the distribution excluding the first moment affect an individual's outcome, consider a thought experiment where a school administrator is trying to decide into which of two (sufficiently large) sections to enroll a new student (whom we refer to as i). Suppose both sections exhibit the same level of average expected effort, $\mathbb{E}_\mu[e] \equiv e_H\mu_H + e_L\mu_L$, but the first one has more students who prefer e_H to e_L and a higher value of μ_H. This can be the case, for instance, when students who prefer e_H are relatively more immune to influence in the second section, and levels of influence offset differences in preferences. That is, due to the lower β-s in the first section, both the probability of choosing the default effort level e^* and μ_H are higher in it than the second section.[30] Then, holding the expectation fixed, $\mu_H + \mu_L$ is lower in the first section with a higher μ_H, as $e_H > e_L$. Relative values of $\mu_H\mu_L$ are ambiguous, but except at small values of μ_H, increasing μ_H does not increase $\mu_H\mu_L$ by enough (and mostly decreases $\mu_H\mu_L$) to offset the decrease in $\mu_H + \mu_L$. When $\varrho \geq 1/2$, the instances of μ_H being very small are sufficiently rare. Accordingly, given that $\mathbb{E}[e_i]$ is greater if $\beta_i(\mu_H + \mu_L) + (1-\beta_i)\mu_H\mu_L$ is smaller, the first section will see a higher value of $\mathbb{E}[e_i]$.

The lesson for the administrator is, that apart from merely looking at the average outcomes of the students in each class, they must assess how many of the new student's peers will truly have a preference for putting in high effort. Between two classes with

[27]In the latter formulation, $\lambda_i \equiv \frac{(n-1)(1-\beta_i)}{n-1+\beta_i}$

[28]While we consider individual-specific regressions, see Angrist (2014) regarding potential problems with peer effect regressions.

[29]$\varrho \geq 1/2$ is naturally the setting in which inducing high effort is desirable ex-ante. Otherwise, we would be acting against the preferences of a majority of students.

[30]The same μ_H leads to a greater $p_i(e_H)$ for a student with a preference for high effort instead of low effort, for any given β_i. Keeping preferences fixed, attention probabilities decline in β_i, thus increasing the probability of e^* being chosen. Together, these two effects can produce the conditions described.

the same average outcomes, the distribution of preferences will determine which section leaves the new student better off. In the example above, they must sort the student into the class with more students who prefer high effort to low effort. On the other hand, without knowledge of the model and the underlying parameters, the administrator cannot accurately determine the tradeoff between a higher average outcome and the form of the preference distribution.

7.2 The social multiplier

Given the nature of non-linear feedback effects observed in our model, a natural question that arises relates to the existence of a social multiplier in the context of a common exogenous shock to fundamentals. Such a shock to fundamentals will naturally have a direct effect on individual outcomes. But, in the presence of social interactions these direct effects may amplify and produce additional indirect equilibrium effects through the mechanism of socially generated attention The social multiplier, defined as the *ratio of the total direct and indirect effects to just the direct effect*, measures the potency of these endogenous social effects. The question about the existence of a social multiplier is an interesting one precisely because it is dependent on the full specification of the nature of social interactions and not just on a reduced form demonstration about the presence of peer effects. That is, models of peer effects implying the same reduced form specification may still differ on whether they exhibit a social multiplier. Key examples are the models of influence through spillovers and conformity, both of which imply a linear-in-means reduced form relationship between individual and peer behavior, though only the former admits a social multiplier.

Once again, we draw on the test taking model to explore the question about a social multiplier under the PIA mechanism. To check for this, however, we must first operationalize what is a common shock, and define precisely, the resultant direct effect. In this section, we look at the case of introducing an additional student i^* to the cohort, one who is a rational decision maker not influenced by others, and prefers e_H to e_L.[31] We want to study the consequence of introducing this student on the behavior of other students. We consider the simple case where there are n incumbent students in the cohort, all of whom have a common immunity from influence parameter β, and k of whom prefer e_H to e_L.

In this set-up, a PIA equilibrium can be fully characterized as a solution to a system of two equations in two unknowns, γ_H and γ_L, which denote the probability of an individual paying attention to e_H and e_L, respectively.[32] The direct effect of the shock is calculated

[31] In other words, the decision-making of this student is outside the considerations in the model and, therefore, introducing her serves as an exogenous common shock.

[32] Note that attention probabilities do not depend on preferences; they only depend on β_i, which are common in this case.

as the change in outcomes when attention probabilities change to

$$\gamma_H^d = \beta + (1-\beta)\left(\frac{k\gamma_H + (n-k)(1-\gamma_L)\gamma_H + 1}{n+1}\right)$$
$$\gamma_L^d = \beta + (1-\beta)\left(\frac{k(1-\gamma_H)\gamma_L + (n-k)\gamma_L}{n+1}\right)$$

where γ_H and γ_L are the attention probabilities under the original equilibrium. The total effect is calculated as the change in outcomes from the original equilibrium when attention probabilities change to γ_H^*, γ_L^*, which underlie the new PIA equilibrium in the presence of the new student choosing e_H with probability 1.

We find that the existence of the social multiplier depends on which outcome variable is being considered. When considering the average probability of choosing either effort level, the model admits a social multiplier for all parameterizations (barring the degenerate $k = n$).[33] On the other hand, the effect of the shock on average expected effort is ambiguous and more nuanced. The following result provides a sufficient condition for the existence of the multiplier.

Proposition 7.1. *Suppose that*

$$\beta \geq \frac{e_L}{\frac{k}{n}e_L + \frac{n-k}{n}e_H}$$

Then the direct and indirect effects on average expected effort are both positive. Consequently, the social multiplier exists.

Proof: Refer to Appendix A.11

The result and its proof consist of some interesting observations. The first is that the shock, though in favor of choosing the high effort level, can actually decrease the average expected effort, both in terms of a direct and total effect, due to an increase in the probability of choosing the default alternative. The sufficient condition is therefore not only ensuring that the social multiplier exists, but that the direct and total effects of the shock are of the intended sign. The second is that the existence of the social multiplier is guaranteed by a condition that requires the peer influence coefficient, $1-\beta$, be small enough, rather than large enough. This highlights the crucial point that the existence of a social multiplier is not a natural corollary of a positive peer effects coefficient. From an empirical point of view, a statistically significant and strong peer effects coefficient should not be interpreted as evidence for the presence of a social multiplier. The social multiplier crucially depends on the nature of peer interactions taking place under a hypothesized mechanism of peer effects. In the case of the PIA model, the non-existence of a

[33]Let $\mu_H(x, y) = \frac{kx+(n-k)x(1-y)}{n}$ be the average probability of choosing e_H when attention probabilities are x and y for high and low effort, respectively. The social multiplier is then $\frac{\mu_H(\gamma_H^*, \gamma_L^*)-\mu_H(\gamma_H, \gamma_L)}{\mu_H(\gamma_H^d, \gamma_L^d)-\mu_H(\gamma_H, \gamma_L)}$. Likewise for other outcomes. See Appendix A.11 for more details.

social multiplier on average effort under some parameterizations points to the potentially sharp increase in the probability of choosing zero effort in the presence of any preference disagreement when β is low.

8 Conclusion

In this paper, we have developed the micro-foundations of a theory of peer effects via the mechanism of socially generated attention. To do so, we introduced the new equilibrium notion of a peer-influenced attention equilibrium. We established that a PIA equilibrium exists for any admissible set of parameters. We then showed that these parameters of the model are uniquely identified. In this exercise, we highlighted how traditional decision-theoretic methods of exploiting inter-menu variations in behavior can be productively used to get at the question of identification, especially when it comes to identifying the underlying social network driving endogenous effects. Our approach, therefore, can complement econometric ones in solving challenging questions of identification in environments with social interactions and peer effects, a strategy that we hope future work will utilize. We also provided three intuitive axioms that characterize the class of datasets that may arise under a PIA equilibrium. This allows us to falsify the model based on choice data, which is important to do as different mechanisms of peer influence, with very different economic and policy implications may be compatible with the same reduced form specification. Finally, we sketched out some important lessons our framework has for the empirical measurement of peer effects. We show that the LIM estimator delivers generally inconsistent estimates of the peer effects coefficient, and provide conditions for the direction of the asymptotic bias. We also find that the existence of a social multiplier depends on the outcome variable of choice, and highlight that stronger, positive peer effects does not automatically imply a social multiplier – in fact, these can move in the opposite direction.

Our work, in its current scope, is focused on providing a formal analysis of endogenous peer effects emanating from socially generated attention. We analyze the PIA mechanism and its theoretical implications for observable behavior, focusing on this endogeneity. Given these foundations, the mechanism can be naturally extended to a quantitative model better suited for a rigorous empirical exercise. The literature on peer effects discusses the challenges of identifying endogenous peer effects when contextual and correlated effects also affect behavior. This paper highlights how one may go about identifying both the sets of peers and the extent of peer effects. However, we do so in an environment where the relationship between an individual's behavior and that of their peers exists solely through the endogenous peer effects channel. Incorporating contextual and correlated effects in our model would allow a full determination of the extent to which our approach complements existing empirical ones to measuring peer effects. In particular, the question of how contextual effects may impact attention appears to be an important one.[34] We hope future work will utilize the framework we have developed here to analyze these questions.

A Appendix

A.1 Key Lemmas

We now prove a few lemmas that hold for PIA equilibria. These lemmas will be used to prove the identification result (Theorem 4.1) and the necessity of the axioms for the representation (Theorem 6.1). Let p be a PIA rationalizable joint random choice rule under parameters $(\{\succ_i: i \in I\}, N, \{\beta_i : i \in I\})$. In the way of notation, for any $i \in I$, $A \in \mathcal{X}$ and $a \in A$, let

$$\mu_{N(i)}(a, A) = \frac{1}{|N(i)|} \sum_{j \in N(i)} p_j(a, A)$$

denote the average probability of choosing a in A in i's cluster $N(i)$. Whenever there is no ambiguity regarding the underlying clustering we are referring to when evaluating this average choice probability, we will simply write μ_i instead of $\mu_{N(i)}$. Further, we say that a non-singleton menu $A \in \mathcal{X}$ is a *full support menu (FSM)* for $i \in I$ if $p_i(a, A) > 0$ for all $a \in A$. We denote the set of all FSMs for i by $\mathcal{X}_i$. Then, the following conclusion follows.

Lemma A.1. *For every $i \in I$, $A \notin \mathcal{X}_i$ iff $\exists a$ such that $a = \max_{\succ_j} A$ for all $j \in N(i)$.*

Proof. Suppose there exist $j, j' \in N(i)$ such that $\max_{\succ_j} A \neq \max_{\succ_{j'}} A$. Then, if $a = \max_{\succ_i} A$, $\exists j \in N(i)$ s.t. $b = \max_{\succ_j} A$, $a \neq b$. Since $\beta_k > 0$, $\forall k \in I$, $p_i(a, A) = \gamma_i(a, A) = \beta_i + (1 - \beta_i)\mu_i(a, A) > 0$, and $p_j(b, A) = \gamma_j(b, A) = \beta_j + (1 - \beta_j)\mu_i(b, A) > 0$. This implies $p_i(a', A) < 1,\ \forall a' \in A \setminus a$; $p_j(a', A) < 1, \forall a' \in A \setminus b$. That is, $\forall a' \in A$, $p_\ell(a', A) < 1$, for some $\ell \in N(i)$. Accordingly, $\mu_i(a', A) < 1,\ \forall a' \in A$.

Now consider any $c \in A$. Since $\mu_i(c, A) < 1$ and $\beta_i \in (0, 1)$, $\gamma_i(c, A) = \beta_i + (1 - \beta_i)\mu_i(c, A) \in (0, 1)$. $\gamma_i(c, A) \in (0, 1)$ implies $1 - \gamma_i(c, A) \in (0, 1)$. As a consequence, $p_i(c, A) = \gamma_i(c, A) \prod_{c' \succ_i c}(1 - \gamma_i(c', A)) \in (0, 1)$. Since this is true for all $c \in A$, $A \in \mathcal{X}_i$.

Next, suppose exists $a \in A$ such that $a = \max_{\succ_j} A$ for all $j \in N(i)$. Suppose $p_i(a, A) < 1$. Then $\mu_i(a, A) < 1$. For all $j \in N(i)$, $\beta_j > 0$ implies

$$p_j(a, A) = \gamma_j(a, A) = \beta_j + (1 - \beta_j)\mu_i(a, A) > \mu_i(a, A)$$

[34]For example, think of educational outcomes of students which are clearly affected by the attention students show in the classroom. Very likely, this attention depends on the classroom infrastructure and quality of the school, which are partly determined by contextual factors like the income of the neighborhood, especially when schools are financed by local taxes.

That is, $\forall j \in N(i), p_j(a, A) > \mu_i(a, A)$, which implies $\mu_i(a, A) > \mu_i(a, A)$! Hence, under a PIA equilibrium, $p_i(a,\, A) = 1$ and $A \notin \mathcal{X}_i$. $\square$

The proof of the Lemma establishes the following corollary.

Corollary A.1. *For $i \in I$, $A \in \mathcal{X}$, and some $a \in A$, the following statements are equivalent:*

1. $A \notin \mathcal{X}_i$
2. $p_i(a, A) = 1$
3. $a = \max_{\succ_j} A$ *for all* $j \in N(i)$
4. $p_j(a, A) = 1, \forall j \in N(i)$

Lemma A.2. *Let $A \in \mathcal{X}$ and $i \in I$. Then:*

(i) $A \in \mathcal{X}_i \implies \left[a = \max_{\succ_i} A \iff \frac{1-p_i(a,A)}{1-p_i(b,A)} < \frac{1-\mu_i(a,A)}{1-\mu_i(b,A)},\ \forall b \in A \setminus a\right]$

(ii) $A \notin \mathcal{X}_i \implies [a = \max_{\succ_i} A \iff p_i(a, A) = 1]$

Proof. Let $A = \{a_1, \ldots, a_M\} \in \mathcal{X}_i$ be s.t. $a_1 \succ_i a_2 \succ_i \cdots \succ_i a_M$. Since $A \in \mathcal{X}_i$, for the derivation below, note that $\sum_{m=1}^{l} p_i(a_m, A) \in (0, 1)$, or equivalently, $1 - \sum_{m=1}^{l} p_i(a_m, A) \in (0, 1)$ for all $l = 1, \ldots, M-1$. Now,

$$p_i(a_1, A) = \gamma_i(a_1, A) = \beta_i + (1 - \beta_i)\mu_i(a_1, A)$$
$$\Rightarrow 1 - \beta_i = \frac{1 - p_i(a_1, A)}{1 - \mu_i(a_1, A)}$$

Note that, under a PIA equilibrium, an alternative $a_l \in A$ not being chosen corresponds to the event that either this alternative is not considered or some alternative preferred to it is considered. Accordingly, it is straightforward to derive that the probability that the $m \succ_i$-top alternatives are not chosen, $1 - \sum_{k \leq m} p_i(a_k,\, A)$, is equal to the probability that none of them is considered, which by the independence of the consideration of alternatives is $\prod_{k \leq m}(1 - \gamma_i(a_k,\, A))$. Then,

$$p_i(a_m,\, A) = \gamma_i(a_m,\, A) \prod_{k \leq m-1} (1 - \gamma_i(a_k,\, A))$$
$$\implies \gamma_i(a_m,\, A) = \frac{p_i(a_m,\, A)}{1 - \sum_{k \leq m-1} p_i(a_k,\, A)}$$
$$\implies 1 - [\beta_i + (1 - \beta_i)\mu_i(a_m,\, A)] = 1 - \frac{p_i(a_m,\, A)}{1 - \sum_{k \leq m-1} p_i(a_k,\, A)}$$

Accordingly,

$$1 - \beta_i = \frac{1 - \frac{p_i(a_m,\, A)}{1 - \sum_{k \leq m-1} p_i(a_k,\, A)}}{1 - \mu_i(a_m,\, A)} \tag{4}$$

Since $1 - \sum_{k\leq m-1} p_i(a_k,\, A) \in (0,\, 1)$ for all $2 \leq m \leq M$,

$$1 - \frac{p_i(a_m,\, A)}{1 - \sum_{k\leq m-1} p_i(a_k,\, A)} < 1 - p_i(a_m,\, A)$$
$$\implies \frac{1 - p_i(a_1,\, A)}{1 - \mu_i(a_1,\, A)} = 1 - \beta_i < \frac{1 - p_i(a_m,\, A)}{1 - \mu_i(a_m,\, A)}$$

Hence, $\frac{1-p_i(a,A)}{1-\mu_i(a,A)} < \frac{1-p_i(b,A)}{1-\mu_i(b,A)}$ or $\frac{1-p_i(a,A)}{1-p_i(b,A)} < \frac{1-\mu_i(a,A)}{1-\mu_i(b,A)}$, $\forall b \in A \setminus a$, if $a = \max_{\succ_i} A$. To establish the only if direction, suppose $\frac{1-p_i(a,A)}{1-p_i(b,A)} < \frac{1-\mu_i(a,A)}{1-\mu_i(b,A)}$, $\forall b \in A \setminus a$, but $\hat{a} \neq a$ is $\succ_i$-best in A. Then based on the argument above, it follows that $\frac{1-p_i(\hat{a},A)}{1-p_i(a,A)} < \frac{1-\mu_i(\hat{a},A)}{1-\mu_i(a,A)}$, or $\frac{1-p_i(a,A)}{1-p_i(\hat{a},A)} > \frac{1-\mu_i(a,A)}{1-\mu_i(\hat{a},A)}$, a contradiction!

Now consider the case $A \notin \mathcal{X}_i$. Lemma A.1 implies that there exists $a \in A$ such that $a = \max_{\succ_j} A$ for all $j \in N(i)$. Then as shown in the proof of that Lemma and stated in Corollary A.1, $a = \max_{\succ_i} A$ iff $p_i(a, A) = 1$. □

Lemma A.2 implies the following corollary:

Corollary A.2. *For all $i \in I$ and $A \in \mathcal{X}$,*

$$a = \max_{\succ_i} A \iff \frac{1 - p_i(a, A)}{1 - p_i(b, A)} \leq \frac{1 - \mu_i(a, A)}{1 - \mu_i(b, A)}, \forall b \in A \setminus a$$

Proof. If $A \in \mathcal{X}_i$, then the conclusion follows immediately from the first statement of Lemma A.2.

Now suppose $A \notin \mathcal{X}_i$. By the second statement of Lemma A.2, $a = \max_{\succ_i} A$ iff $p_i(a,\, A) = 1$ iff $p_i(b,\, A) = 0$, $\forall b \in A \setminus a$. Then, for any $b \in A \setminus a$, $\frac{1-p_i(a,\,A)}{1-p_i(b,\,A)} = \frac{0}{1} = 0$. Further, $p_i(b,\, A) = 0 \implies \mu_i(b,\, A) < 1$. Accordingly, $\frac{1-\mu_i(a,\,A)}{1-\mu_i(b,\,A)}$ is defined and non-negative. Thus, $\frac{1-p_i(a,\,A)}{1-p_i(b,\,A)} \leq \frac{1-\mu_i(a,\,A)}{1-\mu_i(b,\,A)}, \forall b \in A \setminus a$. To establish the other direction, suppose $\frac{1-p_i(a,\,A)}{1-p_i(b,\,A)} \leq \frac{1-\mu_i(a,\,A)}{1-\mu_i(b,\,A)}$, $\forall\, b \in A \setminus a$, but $a' = \max_{\succ_i} A \neq a$. Then, by Lemma A.2, $1 - p_i(a',\, A) = 0$ and $\frac{1-p_i(a,\,A)}{1-p_i(a',\,A)}$ is undefined. Thus, the inequality cannot hold for all $b \in A \setminus a$, leading us to a contradiction. □

The following conclusion follows immediately from the last result.

Corollary A.3. *For all $i \in I$ and $a, b \in X$,*

$$a \succ_i b \iff \frac{1 - p_i(a, ab)}{1 - p_i(b, ab)} \leq \frac{1 - \mu_i(a, ab)}{1 - \mu_i(b, ab)}$$

A.2 Proof of Proposition 2.1

Let $a, b \in A$ be s.t. $a \succ_i b$. If $c = \max_{\succ_j} A$ for all $j \in N(i)$, then it follows that $p_j(c,\, A) = 1$ for all $j \in N(i)$. If $a = c$, then $p_i(a,\, A) = 1 > p_i(b,\, A) = 0$. If $a \neq c$, then by $a \succ_i b$, $b \neq c$, so $p_i(a,\, A) = p_i(b,\, A) = 0$.

If the $\succ_j$-best alternative in A is not common to all $j \in N(i)$, then $p_i(c, A) \in (0, 1)$ for all $c \in A$. Since $\beta_i \in (0, 1)$,

$$\gamma_i(a, A) = \beta_i + (1-\beta_i)\frac{\sum_{j\in N(i)} p_j(a, A)}{|N(i)|} > \beta_i \geq 0.5$$

Then, $\gamma_i(a, A) > 0.5 > 1 - \gamma_i(a, A)$. Since $a \succ_i b$,

$$\begin{aligned}
p_i(b, A) &= \gamma_i(b, A) \prod_{a'\in A, a'\succ_i b} (1-\gamma_i(a', A)) \\
&\leq \prod_{a'\in A, a'\succ_i b} (1-\gamma_i(a', A)) \\
&\leq (1-\gamma_i(a, A)) \prod_{a'\in A,\, a'\succ_i a} (1-\gamma_i(a', A)) \\
&< \gamma_i(a, A) \prod_{a'\in A,\, a'\succ_i a} (1-\gamma_i(a', A)) \\
&= p_i(a, A)
\end{aligned}$$

Since $p_i(a, A) > 0$ if and only if either the $\succ_j$-best alternative in A is not common to all $j \in N(i)$ or $a = \max_{\succ_j} A$ for all $j \in N(i)$, the proof is complete.

A.3 Proof of Proposition 3.1

Fix a cluster of size J and a two-alternative menu $\{a, b\}$. Let

$$S_a := \{i : a \succ_i b\}, \qquad S_b := \{i : b \succ_i a\}.$$

Assume S_a, S_b non-empty. Write

$$x := \frac{1}{J}\sum_{j=1}^{J} p_j(a), \qquad y := \frac{1}{J}\sum_{j=1}^{J} p_j(b)$$

for the cluster-average probabilities of choosing a and b, respectively.

For each individual i, the PIA attention probabilities are

$$\gamma_i(a) = \beta_i + (1-\beta_i)x, \qquad \gamma_i(b) = \beta_i + (1-\beta_i)y.$$

With two alternatives, the top-ranked option is chosen with probability $\gamma_i(\cdot)$, while the lower-ranked option is chosen with probability $\gamma_i(\cdot)(1-\gamma_i(\text{top}))$. Hence:

- If $i \in S_a$, then

$$p_i(a) = \beta_i + (1-\beta_i)x,$$

and

$$p_i(b) = \big(\beta_i + (1-\beta_i)y\big)\big(1 - \beta_i - (1-\beta_i)x\big).$$

- If $i \in S_b$, then

$$p_i(b) = \beta_i + (1-\beta_i)y,$$

and

$$p_i(a) = \big(\beta_i + (1-\beta_i)x\big)\big(1 - \beta_i - (1-\beta_i)y\big).$$

Averaging across the cluster, define

$$A := \frac{1}{J}\sum_{i\in S_a}\beta_i, \qquad B := \frac{1}{J}\sum_{i\in S_a}(1-\beta_i), \qquad C := \frac{1}{J}\sum_{i\in S_b}\beta_i(1-\beta_i), \qquad D := \frac{1}{J}\sum_{i\in S_b}(1-\beta_i)^2,$$

and similarly

$$A' := \frac{1}{J}\sum_{i\in S_b}\beta_i, \qquad B' := \frac{1}{J}\sum_{i\in S_b}(1-\beta_i), \qquad C' := \frac{1}{J}\sum_{i\in S_a}\beta_i(1-\beta_i), \qquad D' := \frac{1}{J}\sum_{i\in S_a}(1-\beta_i)^2.$$

Then the average-choice equations become

$$x = A + Bx + (1-y)(C + Dx),$$

and

$$y = A' + B'y + (1-x)(C' + D'y).$$

Rearranging,

$$x = \phi(y) := \frac{A + C - Cy}{1 - B - D + Dy},$$

and

$$y = \psi(x) := \frac{A' + C' - C'x}{1 - B' - D' + D'x}.$$

Because each $\beta_i \in (0,1)$, we have $(1-\beta_i)^2 < (1-\beta_i)$. Therefore,

$$B + D < \frac{|S_a|}{J} + \frac{|S_b|}{J} = 1, \qquad B' + D' < 1.$$

Hence the denominators are strictly positive on $[0,1]$, so ϕ and ψ are well-defined continuous fractional-linear maps on $[0,1]$.

Now define

$$H(x) := \phi(\psi(x)).$$

Since ϕ and ψ are fractional-linear, so is H, in fact,

$$\begin{aligned} H(x) &= \frac{(A+C)(1-B'-D'+D'x)-C(A'+C'-C'x)}{(1-B-D)(1-B'-D'+D'x)+D(A'+C'-C'x)} \\ &\equiv \frac{a+bx}{c+dx} \end{aligned}$$

Observe that (x, y) is consistent with a PIA equilibrium if and only if $x = H(x)$ and $y = \psi(x)$. Then, $H(x) - x = \frac{q(x)}{c+dx}$, where $q(x)$ has degree at most 2. We have already established earlier conditions which imply that $c + dx \neq 0$. Then, we just need to show that $q(x)$ has a unique root in [0, 1].

Since $\psi(0) = \frac{A'+C'}{1-B'-D'}$ and $A' + B' + C' + D' < 1$, $\psi(0) \in (0, 1)$, implying $H(0) > 0$. Since q has at most two roots, it is enough to show that $H(1) < 1$. Observe that

$$\begin{aligned} H(1) &= \frac{(A+C)(1-B')-CA'}{(1-B')(1-B-D)+DA'} \\ &= \frac{(A+C)(A+B+A')-CA'}{(A+B+A')(A+B'+A'-D)+DA'} \end{aligned}$$

using $A+B+A'+B' = 1$. Further, see that $B'-C = D$. Then, $B'+A'-D-C = A' > 0$. This implies that the first term of the numerator of $H(1)$ is smaller than the first term of the denominator. Since $C, D > 0$, we have $H(1) < 1$.

A.4 Proof of Theorem 3.2

Let $A = \{a_1, ..., a_M\}$ a menu, $N^s = 1, ..., J$ a cluster. Fix an agent i with immunity from influence coefficient $\beta_i \in (0, 1)$. Let $\gamma_r = \beta_i + (1-\beta_i)\mu_r$ and $p_r = \gamma_r \prod_{s<r}(1-\gamma_s)$, where γ_r is the probability that i pays attention to her r-th best alternative, μ_r the average probability of that alternative being chosen in N^s, and p_r is i's probability of choosing it. As described in Section 3, we can define a map $\tilde{\Phi}_i = P_i \Phi_i P_i^{-1}$, where $\Phi_i : \Delta_M \to \Delta_M$ with alternatives order as a_1, ..., a_M and $\tilde{\Phi}_i$ is the same mapping written in a reordered basis, particularly in correspondence with $\succ_i$. Then, $p_r = \tilde{\Phi}_{i,r}(\mu)$, where μ is ordered in i's preference ranking.

Let $Q_r = \prod_{s<r}(1-\gamma_s)$. Then $p_r = \gamma_r Q_r$. Let $K = D_{\tilde{\Phi}}(\mu)$ evaluated at μ. Then

$$K_{r,r} \;=\; (1-\beta_i)\, Q_r, \qquad K_{r,s} \;=\; -\frac{\gamma_r(1-\beta_i)Q_r}{1-\gamma_s} \;\; (s<r), \qquad K_{r,s} \;=\; 0 \;\; (s>r).$$

So K is lower-triangular in this basis, with non-negative diagonal and non-positive sub-diagonal entries.

Summing absolute values across row r:

$$R_r(\mu) \;:=\; \sum_{s=1}^{M} |K_{r,s}| \;=\; (1-\beta_i)Q_r\Big[1 + \gamma_r \sum_{s<r} \frac{1}{1-\gamma_s}\Big].$$

Writing $q_s := 1-\gamma_s$, the identities $Q_r = \prod_{s<r} q_s$ and $Q_r \sum_{s<r} q_s^{-1} = \sum_{s<r}\prod_{s'<r, s'\neq s} q_{s'} = e_{r-2}(q_1,\dots,q_{r-1})$ (where e_k is the k-th elementary symmetric polynomial) give

$$R_r(\mu) \;=\; (1-\beta_i)\big[\, e_{r-1}(q_1,\dots,q_{r-1}) \;+\; \gamma_r\, e_{r-2}(q_1,\dots,q_{r-1})\,\big]. \tag{5}$$

with the boundary cases

$$\begin{aligned} R_1(\mu) &= 1-\beta_i \\ R_2(\mu) &= (1-\beta_i)\,[q_1+\gamma_2] \end{aligned}$$

Observe that $R_r(\mu)$ achieves a maximum when $\mu_r = 1$, and $g(r) \equiv \sup_{\mu\in\Delta_M} R_r(\mu) = (1-\beta_i)^{r-1}(r-\beta_i)$. Clearly, $g(1) = (1-\beta_i) < (1-\beta_i)(2-\beta_i) = g(2)$. However, for $r \geq 2$, $g(r) \geq g(r+1)$ iff

$$\begin{aligned} \frac{g(r+1)}{g(r)} &= (1-\beta_i)\frac{r+1-\beta_i}{r-\beta_i} \\ &\leq 1 \end{aligned}$$

This rearranges to $\beta_i^2 - (r+1)\beta_i + 1 \leq 0$. Since this is convex in β_i, this region occurs between the two roots. The roots of this quadratic are

$$\beta_{\pm}^{(r)} = \frac{r+1 \pm \sqrt{(r+1)^2-4}}{2}$$

Since the products of the two roots are 1,

$$\beta_{-}^{(r)} = \frac{2}{r+1+\sqrt{(r+1)^2-4}}$$

$\beta_-^{(r)}$ is decreasing in r, and $\beta_-^{(2)} = \frac{3-\sqrt{5}}{2}$. Then, if $\beta_i > \frac{3-\sqrt{5}}{2}$, $g(r) < g(2)$ for all $r \neq 2$. Further, $g(2) = (1-\beta_i)(2-\beta_i) < 1$ whenever $\beta_i \in \left(\frac{3-\sqrt{5}}{2}, 1\right)$. Hence, $||K||_\infty \leq g(2) < 1$.

Therefore, $||D_{\Phi_i}||_\infty < 1$ at all $\mu \in \Delta_M$. By the triangle inequality, $||D_F||_\infty \leq \frac{1}{J}\sum_{i=1}^{J} ||D_{\Phi_i}||_\infty < 1$. Hence, F is a contraction, and has a unique fixed point.

A.5 Proof of Proposition 3.2

By Theorem 3.1 we know that F has a fixed point. By Lemma A.1, we know that any fixed point μ^* of F lies in $\Omega = \mathrm{int}(\Delta_M)$.

By supposition, $\rho(D_F(\mu^*)) < 1$ at any fixed point. Then, $I - D_F(\mu^*)$ is non-singular. Since $D_F(\mu^*)$ is a real matrix, $\det(I - D_F(\mu^*)) > 0$. Then, the mapping $G(\mu) = F(\mu) - \mu$ is locally invertible, implying that fixed points of F are isolated. Denote F's fixed points by μ_1^*, ..., μ_K^*. Further, $0 \notin G(\partial\Delta_M)$ by Lemma A.1, where ∂S refers to the boundary of S. Hence, the global Brouwer degree of G, $\deg(G,\, \Omega,\, 0) = \sum_{i=1}^{K} \text{sign}(\det(D_G(\mu_i^*)))$ is well defined.[35]

To compute the degree, we apply the property of *homotopy invariance*. We construct a continuous deformation from $G(\mu)$ to a simple, analytically tractable linear map. Let $\mu_0 \in \Omega$ be some arbitrarily chosen point (for example, the barycenter where $\mu_{0,m} = \frac{1}{M+1}$ for all m). We define the simple linear base mapping $G_0 : \Delta_M \to \mathbb{R}^M$ as:

$$G_0(\mu) = \mu_0 - \mu \tag{6}$$

We now define a linear homotopy $H : [0, 1] \times \overline{\Omega} \to \mathbb{R}^M$ connecting G and G_0:

$$H(t, \mu) = (1 - t)G(\mu) + tG_0(\mu) \tag{7}$$

Substituting the definitions of $G(\mu)$ and $G_0(\mu)$ yields:

$$\begin{aligned} H(t, \mu) &= (1 - t)(F(\mu) - \mu) + t(\mu_0 - \mu) \\ &= (1 - t)F(\mu) - (1 - t)\mu + t\mu_0 - t\mu \\ &= (1 - t)F(\mu) + t\mu_0 - \mu \end{aligned} \tag{8}$$

To establish that the Brouwer degree is invariant under this deformation, we must prove that the homotopy is *admissible*, meaning it never vanishes on the boundary for any parameter $t \in [0, 1]$. That is, $H(t, \mu) \neq 0$ for all $(t, \mu) \in [0, 1] \times \partial\Omega$. For $t = 0$, $H(t,\, \mu) = G(\mu)$, so it is non-zero on $\partial\Omega$. For $t = 1$, we get $\mu_0 - \mu$, which is non-zero on $\partial\Omega$ by construction. For $t \in (0,\, 1)$, observe that $\mu_0 \in \Omega$ implies $(1 - t)F(\mu) + t\mu_0 \in \Omega$. Then, $H(t,\, \mu) \neq 0$ on $\partial\Omega$. Hence, $\deg(G,\, \Omega,\, 0) = \deg(G_0,\, \Omega,\, 0)$.

Observe that $D_{G_0}(\mu) = -I$, and G_0 has the lone root, μ_0. Hence, $\deg(G_0,\, \Omega,\, 0) = (-1)^M$. We already proved that $\det(I - D_F(\mu^*)) > 0$, which means $\text{sign}(\det(D_F(\mu^*) - I)) = (-1)^M$. Then, by homotopy invariance, we have

$$(-1)^M = K(-1)^M$$

implying $K = 1$. Therefore, G has a unique root and F has a unique fixed point.

[35] We use the topological definition of degree, so we do not have to worry even if 0 is a critical value of G.

A.6 Numerical support in favor of uniqueness

By Proposition 3.2, a sufficient condition for PIA equilibria to be unique is

$$\rho(D_F(\mu^*)) < 1 \tag{spec}$$

at any fixed point μ^* of F. By Theorem 3.2, we know that spec holds when $\beta_{\min} > \frac{3-\sqrt{5}}{2}$. As examples in Section 3 highlight, $\rho(D_F)$ is not less than 1 uniformly over Δ_M. Since $\rho(|A|) \geq \rho(A)$ for all real matrices A, we check the stronger condition

$$\rho(|D_F(\mu^*)|) < 1 \tag{spec*}$$

Since we know the analytical form of the Jacobian, we verify this condition using the following steps

1. For each $(|N^s|, M, \vec{\beta}, \succ)$, run damped iteration of F from multiple random Dirichlet starting points.
2. Cluster the converged iterates with ℓ^∞ tolerance.
3. Refine each clustered iterate by Newton's method applied to the residual $F(\mu) - \mu = 0$, to machine precision.
4. Verify the refined point lies in the strict interior of Δ_M (stringent margin).
5. Compute $\rho(|J_F(\mu^*)|)$ at each verified fixed point.

Step 3–5 outputs a (possibly empty, possibly multi-element) *list* of fixed points per configuration. Uniqueness is never assumed: if a configuration has multiple distinct interior fixed points, the procedure could return multiple of them and verify the bound at each. As a robustness check, use a Newton method based solver and symbolic polynomial solver to supplement iteration based results. Building on the characteristics of the examples in Section 3, we run the following searches.

Broad sweep. Each trial draws $(|N^s|, M, \vec{\beta}, \succ)$ from joint distributions designed to oversample the harder regime:

- $(|N^s|, M)$: $|N^s| \in \{2, \ldots, 15\}$, $M \in \{2, \ldots, 7\}$.
- $\vec{\beta}$: mixture of log-uniform, "one moderate / rest tiny", "one tiny / rest moderate", and "all tiny".
- $\succ$: mixture of random, one-vs-rest, two opposing camps, unanimous-except-one, and block-structured preferences.

Danger-zone sweep. Focused on two-camp preference structures with $\vec{\beta}$ drawn from extreme small-β distributions (log-uniform on $(10^{-12}, 10^{-3})$). This is the regime where the broad sweep identified ρ approaching 1.

A.6.1 Results

After refinement, a total of 2,306 verified interior fixed points were evaluated: 2,196 from the broad sweep and 110 from the danger-zone sweep. The headline statistics:

quantity	value
# interior fixed points with $\rho(\lvert J_F(\mu^*)\rvert) \geq 1$	**0**
# interior fixed points with $\rho \geq 0.9999$	55
# interior fixed points with $\rho \geq 0.999$	164
# interior fixed points with $\rho \geq 0.99$	377
maximum ρ observed (any configuration)	0.99999999999630
gap $1 - \rho$ at the worst case	3.7×10^{-12}
# configurations with > 1 interior fixed point	**0**

No fixed point violates the spectral condition. Equally important: *no configuration in the sweep, after refinement, is found to have multiple distinct interior fixed points* – consistent with the prediction of Proposition 3.2.

A note on initial-pass artefacts. The original sweep (before refinement to machine precision) appeared to report 11 configurations with multiple distinct fixed points in the danger-zone regime ($\beta_{\min} \leq 10^{-9}$). Re-running each "reported" fixed point through Newton's method to residual 10^{-15} revealed that the apparent multiplicity in every case was a numerical artefact: either the multiple "fixed points" collapsed under refinement to the same point (clustering tolerance too loose for the near-boundary regime), or the additional "fixed points" refined to non-converged iterates (residuals $\sim 10^{-5}$). After refinement, we find exactly one verified interior fixed point in each configuration.

Dependence on $\beta_{\min}$.

$\beta_{\min}$ bin	# samples	max ρ	99.5% quantile of ρ
$[10^{-1}, 0.5)$	186	0.881056	0.853439
$[10^{-2}, 10^{-1})$	281	0.973886	0.964324
$[10^{-3}, 10^{-2})$	144	0.993954	0.992320
$[10^{-4}, 10^{-3})$	192	0.999876	0.999808
$[10^{-5}, 10^{-4})$	265	0.999978	0.999956
$[10^{-6}, 10^{-5})$	419	0.999839	0.999736
$[10^{-7}, 10^{-6})$	730	0.999914	0.999875
$[10^{-8}, 10^{-7})$	10	$\approx 1 - 10^{-8}$	—
$[10^{-10}, 10^{-8})$	75	$\approx 1 - 10^{-10}$	—
$[0, 10^{-10})$	4	$\approx 1 - 10^{-12}$	—

A.7 Proof of Theorem 4.1

Let $\mathcal{X}$ such that $\{a, b\} \in \mathcal{X}$ for all $a, b \in X$. Suppose p is a dataset that can be PIA rationalized by $\xi = \left\{(\succ_i)_{i\in I}, N, (\beta_i)_{i\in I}\right\}$ and $\hat{\xi} = \left\{(\hat{\succ}_i)_{i\in I}, \hat{N}, \left(\hat{\beta}_i\right)_{i\in I}\right\}$. We will show that $\xi = \hat{\xi}$.

Before proving the main result, we prove an intermediate step that allows us to simplify the problem.

Lemma A.3. *Suppose p is a PIA rationalizable dataset. For all $A \in \mathcal{X}$, $p_i(a, A) = 1$ iff $p_i(a, ab) = 1$ for all $b \in A \setminus \{a\}$.*

Proof. Let ξ be a collection of parameters that PIA rationalize p. By Lemma A.1, $p_i(a, A) = 1$ iff $a = \max_{\succ_j} A$ for all $j \in N(i)$ iff $a \succ_j b$ for all $b \in A \setminus \{a\}$ and $j \in N(i)$. Again, by Lemma A.1, this is equivalent to $p_i(a, ab) = 1$ for all $b \in A \setminus \{a\}$. □

For each $i \in I$, define $R(i)$ as

$$\begin{aligned} R(i) &\equiv \{j \in I : p_i(a, A) = 1 \iff p_j(a, A) = 1, A \in \mathcal{X}, a \in A\} \\ &\equiv \{j \in I : p_i(a, ab) = 1 \iff p_j(a, ab) = 1, \{a, b\} \in \mathcal{X}\} \end{aligned}$$

Lemma A.3 establishes these two definitions as equivalent.

<u>**Uniqueness of Clusters**</u>: By Corollary A.1, $p_i(a, ab) = 1 \iff p_j(a, ab) = 1, \forall j \in N(i)$. Thus, $j \in N(i) \implies j \in R(i)$. Now suppose $j \notin N(i)$. Since the clustering is homophilous, there exists a pair of alternatives $\{a, b\}$ such that a is preferred to b for everyone in one of $N(i)$ and $N(j)$ but not for the other. Suppose w.l.o.g. that $a \succ_{i'} b$ for all $i' \in N(i)$. Then, there exists $j' \in N(j)$ such that $b \succ_{j'} a$. By Corollary A.1, $p_i(a, ab) = 1$ and $p_j(a, ab) \neq 1$. Thus, $j \notin R(i)$. The same argument can be applied if $N(j)$ preferred a over b. Since $j \notin N(i) \implies j \notin R(i)$, $R(i) \subseteq N(i)$, and $N(i) = R(i)$ for all $i \in I$.

Since the same argument applies to $\hat{N}$, $N(i) = R(i) = \hat{N}(i)$ for all $i \in I$, which means that $\hat{N} = N$.

Uniqueness of Preferences: Since $N(i) = \hat{N}(i)$, $\mu_{N(i)}(a, A) = \frac{\sum_{j\in N(i)} p_j(a, A)}{|N(i)|} = \frac{\sum_{j\in \hat{N}(i)} p_j(a, A)}{|\hat{N}(i)|} = \mu_{\hat{N}(i)}(a, A)$ for all $a \in A$, $A \in \mathcal{X}$. Then, by Corollary A.3,

$$a \succ_i b \iff \frac{1 - p_i(a, ab)}{1 - p_i(b, ab)} \le \frac{1 - \mu_{N(i)}(a, ab)}{1 - \mu_{N(i)}(b, ab)} = \frac{1 - \mu_{\hat{N}(i)}(a, ab)}{1 - \mu_{\hat{N}(i)}(b, ab)} \iff a \hat{\succ}_i b$$

Thus, $\succ_i = \hat{\succ}_i$ for all $i \in I$.

Uniqueness of β_i: Since we assume that not everyone's preferences within a cluster are identical, for all $i \in I$, there exists $j, j' \in N(i)$ such that $a \succ_{j'} b$ but $b \succ_j a$, which holds true iff $a \hat{\succ}_{j'} b$ and $b \hat{\succ}_j a$. From Corollary A.1, $p_i(a, ab) \in (0, 1)$, which implies that $\mu_{N(i)}(a, ab) = \mu_{\hat{N}(i)}(a, ab) \in (0, 1)$. Putting everything together, we have

$$\begin{aligned} p_i(a, ab) &= \beta_i + (1 - \beta_i)\mu_{N(i)}(a, ab) \\ &= \hat{\beta}_i + (1 - \hat{\beta}_i)\mu_{\hat{N}(i)}(a, ab) \\ \implies \beta_i &= 1 - \frac{1 - p_i(a, ab)}{1 - \mu_{N(i)}(a, ab)} = 1 - \frac{1 - p_i(a, ab)}{1 - \mu_{\hat{N}(i)}(a, ab)} = \hat{\beta}_i \end{aligned}$$

Thus, $\beta_i = \hat{\beta}_i$ for all $i \in I$.

Thus, we have shown that $\xi = \hat{\xi}$, and the parameters that PIA rationalize a dataset p are uniquely identified.

A.8 Proof of Theorem 6.1

Necessity: Suppose p is a PIA rationalizable joint random choice rule, under parameters ξ, a tuple of strict preference rankings $\{\succ_i : i \in I\}$, homophilous clustering $N = \{N^1, \dots, N^S\}$, and immunity from influence coefficients $\{\beta_i : i \in I\}$. Then p satisfies:

Peer Influence: From Corollary A.1, for any menu A, $a \in A$, $p_i(a, A) = 1 \iff A$ is top-agreeable for $N(i)$ with top $a \iff p_j(a, A) = 1, \forall j \in N(i)$. Then any $j \in N(i)$ is connected to i. Since $|N(i)| > 1$, there exists $j \in N(i)$, $j \neq i$, such that j and i are connected.

Since for each cluster N^s, there exist i, j such that $\succ_i \neq \succ_j$, there exists $a, b \in X$ such that $a \succ_i b$ and $b \succ_j a$. $\{a, b\} \in \mathcal{X}$ by supposition, so $\{a, b\}$ is a menu in which individuals in N^s choose stochastically by Lemma A.1.

Stochastic IIA: We have established in the proof of Theorem 4.1 that for any PIA rationalizable choice rule with homophilous clustering $N = \{N^1, \dots, N^S\}$, $N(i) = R(i)$, for any $i \in I$. Accordingly, $\mu_{R(i)}(a, A) = \mu_{N(i)}(a, A)$, for any $A \in \mathcal{X}$. Consider any such menu A. Given that $\succ_i$ is a ranking, there exists a unique $a \in A$ such that $a = \max_{\succ_i} A$.

By Corollary A.2, it follows that $\frac{1-p_i(a,A)}{1-p_i(b,A)} \leq \frac{1-\mu_{N(i)}(a,A)}{1-\mu_{N(i)}(b,A)}, \forall b \in A \setminus a$. Further, since $\mu_{N(i)} = \mu_{R(i)}$, we can conclude that $\exists!\ a \in A$ such that $\frac{1-p_i(a,A)}{1-p_i(b,A)} \leq \frac{1-\mu_{R(i)}(a,A)}{1-\mu_{R(i)}(b,A)}, \forall b \in A \setminus a$.

Next, consider any $B \subseteq A$ with $a \in B$. Clearly $a = \max_{\succ_i} B$, and it follows from Corollary A.2 and $\mu_{N(i)} = \mu_{R(i)}$ that $\frac{1-p_i(a,B)}{1-p_i(b,B)} \leq \frac{1-\mu_{R(i)}(a,B)}{1-\mu_{R(i)}(b,B)}, \forall b \in B \setminus a$.

Menu Independence of Influence: Let $A = \{a_1, \ldots, a_M\} \in \mathcal{X}$ be a menu in which i chooses stochastically. Suppose $a_1 \succ_i a_2 \succ_i \cdots \succ_i a_M$. Then $\bar{A}_i(a_1) = \emptyset$ and $\bar{A}_i(a_m) = \{a_1, \ldots, a_{m-1}\}$, for $m = 2, \ldots, M$. This is because by Corollary A.3 and the fact that $\mu_{R(i)}(.) = \mu_{N(i)}(.)$, we know that for any $a, b \in A$, $\frac{1-p_i(a,ab)}{1-p_i(b,ab)} \leq \frac{1-\mu_{R(i)}(a,ab)}{1-\mu_{R(i)}(b,ab)}$ iff $a \succ_i b$. Since all binary menus are contained in $\mathcal{X}$, we can confirm the above claim. Accordingly, $p_i^*(a_1, A) = p_i(a_1, A)$, and

$$p_i^*(a_m, A) = \frac{p_i(a_m, A)}{1 - p_i(a_1, A) - p_i(a_2, A) - \cdots - p_i(a_{m-1}, A)}, \forall m = 2, \ldots, M$$

Further, drawing on equation (4) that we established in the proof of Lemma A.2 and the fact that $\mu_{R(i)}(.) = \mu_{N(i)}(.)$, it follows that for any such A,

$$\frac{1 - p_i(a_1, A)}{1 - \mu_{R(i)}(a_1, A)} = \frac{1 - \frac{p_i(a_2,A)}{1-p_i(a_1,A)}}{1 - \mu_{R(i)}(a_2, A)} = \cdots = \frac{1 - \frac{p_i(a_M,A)}{1-p_i(a_1,A)-p_i(a_2,A)-\cdots-p_i(a_{M-1},A)}}{1 - \mu_{R(i)}(a_M, A)} = 1 - \beta_i$$

i.e.,

$$\frac{1 - p_i(a_1, A)}{1 - \mu_{R(i)}(a_1, A)} = \frac{1 - p_i^*(a_1, A)}{1 - \mu_{R(i)}(a_1, A)} = \frac{1 - p_i^*(a_2, A)}{1 - \mu_{R(i)}(a_2, A)} = \cdots = \frac{1 - p_i^*(a_M, A)}{1 - \mu_{R(i)}(a_M, A)} = 1 - \beta_i < 1$$

Accordingly, for any $A, B \in \mathcal{X}$ in which i chooses stochastically, and any $a \in A$, $b \in B$, we have

$$\frac{1 - p_i^*(a, A)}{1 - \mu_{R(i)}(a, A)} = \frac{1 - p_i^*(b, B)}{1 - \mu_{R(i)}(b, B)} < 1$$

Sufficiency: Let p be a joint random choice rule that satisfies Peer Influence, stochastic IIA, and menu independence of influence. To show that p is PIA rationalizable we need to identify a collection of parameters $\xi \in \Xi$ such that p is a PIA equilibrium represented by ξ.

Defining the clustering: For any $i \in I$, define

$$N(i) = R(i) = \{j \in I : i \text{ and } j \text{ are connected}\}$$

We want to show that this produces a valid partition. First, note that by the symmetry in the definition, $i \in R(j) \iff j \in R(i)$. If $j \in R(i)$, then $p_{j'}(a, A) = 1 \iff p_j(a, A) = 1 \iff p_i(a, A) = 1 \iff p_{i'}(a, A) = 1$, for $j' \in R(j)$ and $i' \in R(i)$. That is, $R(i) = R(j)$. By Peer Influence, $\exists j \neq i$ s.t. $j \in R(i)$, implying that $|N(i)| > 1$.

Finally, suppose $R(i) \neq R(j)$. By the above argument, $j \notin R(i)$. Furthermore, if

$j' \in R(i) \cap R(j)$, by the same argument, $R(i) = R(j') = R(j)$, which is a contradiction. Trivially, $\cup_{i \in I} R(i) = I$, which implies that $\{N(i)\}_{i \in I}$ is a valid clustering.

We next define preferences, and show at the end of the proof that this definition is consistent with N being a homophilous clustering. Since $N(i)$ is defined as $R(i)$ in the following, $\mu_i = \mu_{R(i)} = \mu_{N(i)}$ henceforth.

Defining preferences: Consider $i \in I$ and define $\succ_i \subseteq X \times X$ by: for any $a, b \in X, a \neq b, a \succ_i b$ if $\frac{1-p_i(a,ab)}{1-p_i(b,ab)} \leq \frac{1-\mu_i(a,ab)}{1-\mu_i(b,ab)}$. These preferences are well-defined because $\{a, b\} \in \mathcal{X}$ for all $a, b \in X$. First, we establish that $\succ_i$ is total. This follows from stochastic IIA, since for menu $\{a, b\}$, either $\frac{1-p_i(a,ab)}{1-p_i(b,ab)} \leq \frac{1-\mu_i(a,ab)}{1-\mu_i(b,ab)}$ or $\frac{1-p_i(b,ab)}{1-p_i(a,ab)} \leq \frac{1-\mu_i(b,ab)}{1-\mu_i(a,ab)}$. Hence, we have $a \succ_i b$ or $b \succ_i a$. Since, by stochastic IIA, this inequality holds for a unique alternative, it establishes that $\succ_i$ is asymmetric. Finally, to show that $\succ_i$ is transitive, let $a \succ_i b, b \succ_i c$ and consider $A = \{a, b, c\}$. $A \in \mathcal{X}$ because $\mathcal{X}$ contains all three-alternative menus. By stochastic IIA, $\exists!\, d \in A$ such that $\frac{1-p_i(d, A)}{1-p_i(e, A)} \leq \frac{1-\mu_i(d, A)}{1-\mu_i(e, A)}$ and $\frac{1-p_i(d, de)}{1-p_i(e, de)} \leq \frac{1-\mu_i(d, de)}{1-\mu_i(e, de)}$, for all $e \in A \setminus d$. By the way $\succ_i$ is defined, given that $a \succ_i b$ and $b \succ_i c$, $d \neq b, c$. Hence, $d = a$, and consequently $a \succ_i c$.

To ensure that for each cluster N^s, there exists i, j s.t. $\succ_i \neq \succ_j$, note that there exists some A such that $i \in N^s$ chooses stochastically by Peer Influence. By definition, $N^s = R(i)$, so all $j \in N^s$ choose stochastically in A. Let a be the unique alternative such that $\frac{1-p_i(a, A)}{1-p_i(a', A)} \leq \frac{1-\mu_i(a, A)}{1-\mu_i(a', A)}$. Since i chooses stochastically, there exists another alternative b such that $p_i(b, A) \neq 0$. Then $\mu_i(a, A) \neq 1$, and we can rewrite the stochastic IIA statement as $\frac{1-p_i(a, A)}{1-\mu_i(a, A)} < \frac{1-p_i(a', A)}{1-\mu_i(a', A)}$. However, it cannot be true that this holds with respect to the same alternative, a, for all $j \in N^s$, because $\frac{1}{|N^s|} \frac{\sum_{j \in N^s} 1-p_j(a, A)}{1-\mu_i(a, A)} = \frac{1}{|N^s|} \frac{\sum_{j \in N^s} 1-p_j(a', A)}{1-\mu_i(a', A)} = 1$ for all $a, a' \in A$. Hence, for some $j \in N^s$, there exists $b \neq a$ such that $\frac{1-p_j(b, A)}{1-\mu_j(b, A)} < \frac{1-p_j(a', A)}{1-\mu_j(a', A)}$ for all $a' \neq b$. By stochastic IIA, the same must hold true for the menu $\{a, b\}$. Then, by the definition of preferences, $a \succ_i b$ and $b \succ_j a$.

Defining β_i-s: For any $i \in I$, define β_i as follows by taking any menu $A \in \mathcal{X}$ in which i chooses stochastically, and some $a \in A$:

$$\beta_i = \frac{p_i^*(a, A) - \mu_i(a, A)}{1 - \mu_i(a, A)}$$

Since $1 - \beta_i = \frac{1-p_i^*(a,A)}{1-\mu_i(a,A)}$, menu independence of influence guarantees that the definition of β_i is independent of the choice of a and A, and that $1 - \beta_i < 1$, or $\beta_i > 0$. By how preferences are defined, $b \succ_i a$ iff $b \in \bar{A}_i(a)$, $a, b \in A$. Then, for $a' = \max_{\succ_i} A$, since $\bar{A}_i(a') = \emptyset$, $p_i^*(a', A) = \frac{p_i(a', A)}{1-\sum_{a \in \bar{A}_i(a')} p_i(a, A)} = p_i(a', A)$. Since i chooses stochastically in A, $p_i(a', A) < 1$, which implies $\beta_i = \frac{p_i(a', A) - \mu_i(a', A)}{1-\mu_i(a', A)} < 1$.

Establishing the representation: Consider any $i \in I$. If i chooses non-stochastically in A, then there exists $a \in A$ such that $p_i(a, A) = 1 \iff p_j(a, A) = 1$ for all

$j \in R(i) = N(i)$. Then:

$$\begin{aligned}\gamma_i(a,\, A) &=: \beta_i + (1-\beta_i)\frac{1}{|N(i)|}\sum_{j\in N(i)} p_j(a,\, A)\\ &= \beta_i + 1 - \beta_i\\ &= 1\\ &= p_i(a,\, A)\end{aligned}$$

Note that $\frac{1-p_i(a,\, A)}{1-p_i(b,\, A)} = 0 = \frac{1-\mu_i(a,\, A)}{1-\mu_i(b,\, A)}$, for all $b \in A \setminus a$. By stochastic IIA, this implies that $\frac{1-p_i(a,\, ab)}{1-p_i(b,\, ab)} \leq \frac{1-\mu_i(a,\, ab)}{1-\mu_i(b,\, ab)}$, for all $b \in A \setminus a$, which implies by the definition of $\succ_i$ that $a = \max_{\succ_i} A$. Then, for all $b \in A \setminus a$

$$\begin{aligned}\gamma_i(b,\, A)\prod_{c\succ_i b;\, c\in A}(1-\gamma_i(c,\, A)) &= \gamma_i(b,\, A)(1-\gamma_i(a,\, A))\prod_{c\succ_i b; c\in A\setminus a}(1-\gamma_i(c,\, A))\\ &= 0\\ &= p_i(b,\, A)\end{aligned}$$

Hence, the representation holds for such A.

Now, consider $A = \{a_1,\, ...,\, a_M\}$ in which i chooses stochastically, and assume wlog that $a_1 \succ_i a_2 \succ_i ... \succ_i a_M$. By the definition of $\succ_i$, $\bar{A}_i(a_1) = \emptyset$ and $\bar{A}_i(a_m) = \{a_1,\, ...,\, a_{m-1}\}$ for $m \in \{2,\, ...,\, M\}$. By menu independence of influence,

$$\beta_i = \frac{p_i^*(a_1,\, A) - \mu_i(a_1,\, A)}{1-\mu_i(a_1,\, A)} = \frac{p_i(a_1,\, A) - \mu_i(a_1,\, A)}{1-\mu_i(a_1,\, A)},$$

which implies

$$p_i(a_1,\, A) = \beta_i + (1-\beta_i)\mu_i(a_1,\, A) =: \gamma_i(a_1,\, A)$$

Again applying menu independence of influence, we have that for any a_m,

$$\begin{aligned}&\frac{p_i^*(a_m, A) - \mu_i(a_m, A)}{1-\mu_i(a_m, A)} = \beta_i\\ &\qquad \implies p_i^*(a_m,\, A) = \beta_i + (1-\beta_i)\mu_i(a_m,\, A) =: \gamma_i(a_m,\, A)\end{aligned}$$

Accordingly,

$$\frac{p_i(a_m,\, A)}{1-\sum_{k\leq m-1} p_i(a_k,\, A)} = \gamma_i(a_m,\, A) \tag{9}$$

Next, we establish by induction that $1 - \sum_{k\leq \ell} p_i(a_k,\, A) = \prod_{k\leq \ell}(1-\gamma_i(a_k,\, A))$. To do so, first, we know from above that $1 - p_i(a_1,\, A) = 1 - \gamma_i(a_1,\, A)$. For the inductive step, assume that the equality we want to establish holds for $\ell - 1$, i.e., $1 - \sum_{k\leq \ell-1} p_i(a_k,\, A) =$

$\prod_{k\leq \ell-1}(1-\gamma_i(a_k,\, A))$. Then use equation (9) to get

$$\begin{aligned}
1-\gamma_i(a_\ell,\, A) &= 1-\frac{p_i(a_\ell,\, A)}{1-\sum_{k\leq \ell-1} p_i(a_k,\, A)}\\
&= \frac{1-\sum_{k\leq \ell} p_i(a_k,\, A)}{1-\sum_{k\leq \ell-1} p_i(a_k,\, A)}\\
&= \frac{1-\sum_{k\leq \ell} p_i(a_k,\, A)}{\prod_{k\leq \ell-1}(1-\gamma_i(a_k,\, A))}\\
\implies \prod_{k\leq \ell}(1-\gamma_i(a_k,\, A)) &= 1-\sum_{k\leq \ell} p_i(a_k,\, A)
\end{aligned}$$

So, this expression holds for ℓ, establishing the claim. Accordingly, equation (9) implies that

$$p_i(a_m,\, A) = \gamma_i(a_m,\, A)\prod_{k\leq m-1}(1-\gamma_i(a_k,\, A)),$$

for all $m \in \{2,\, ...,\, M\}$. Hence, the representation holds for all $A \in \mathcal{X}$, and $i \in I$.

Verifying that the clustering is homophilous: We know that $N(i) = R(i)$ by the definition of the clusters. Let N^s and N^t be two distinct clusters, with $i \in N^s$ and $j \in N^t$. Then, $N^s = R(i)$ and $N^t = R(j)$, with $R(i) \neq R(j)$. Since i and j are not connected, there exists a menu A such that $p_i(a,\, A) = 1$ or $p_j(a,\, A) = 1$ for some $a \in A$, but not both. Suppose w.l.o.g. that $p_i(a,\, A) = 1$. Then, $p_{i'}(a,\, A) = 1$ for all $i' \in N^s = R(i)$. In establishing the representation, we showed that $p_{i'}(a,\, A) = 1$ implies $a = \max_{\succ_{i'}} A$. We now want to show that $\max_{\succ_{j'}} A \neq a$ for some $j' \in R(j)$.

Choose $j' \in R(j)$ such that $p_{j'}(a,\, A) \leq \mu_j(a,\, A)$. Of course, such a j' must exist. Furthermore, $p_j(a,\, A) \neq 1 \implies \mu_j(a,\, A) < 1$. If $a = \max_{\succ_{j'}} A$, then based on the fact that choice probabilities are according to the representation as shown above,

$$\begin{aligned}
p_{j'}(a,\, A) &= \gamma_{j'}(a,\, A)\\
&= \beta_{j'} + (1-\beta_{j'})\mu_j(a,\, A)\\
&> \mu_j(a,\, A)
\end{aligned}$$

However, this is not true by assumption, which implies that $a \neq \max_{\succ_{j'}} A$. Then, there exists $b \in A\setminus a$ such that $b \succ_{j'} a$, even as $a \succ_{i'} b$ for all $i' \in N^s$. Therefore, the homophily condition is satisfied.

A.9 Identification: Examples

Example A.1. Suppose $X = \{a,\, b,\, c\}$ is the set of alternatives and $I = \{1,\, 2,\, 3,\, 4\}$ the set of individuals in society. Let the profile of non-trivial random choice rules for these four individuals be given by $p_i(a,\, A) = 1$ for $A = \{a,b\}, \{a,c\}, \{a,b,c\}$, $i \in I$, and for the menu $\{b, c\}$ choice probabilities given by the following:

	p_1	p_2	p_3	p_4
b	$1/2$	$3/5$	$2/5$	$1/2$
c	$\frac{31-\sqrt{805}}{26} \approx 0.101$	$\frac{150-4\sqrt{805}}{481} \approx 0.076$	$\frac{78-3\sqrt{341}}{67} \approx 0.337$	$\frac{457-15\sqrt{341}}{1139} \approx 0.158$

If this choice data has to be consistent with the interactions underlying a PIA equilibrium, then note first that for any menu where an individual chooses stochastically,

$$\gamma_i(a,\, A) = \beta_i + (1 - \beta_i)\mu_{N(i)}(a,\, A) > \mu_{N(i)}(a,\, A),$$

and their best alternative in the menu is chosen with the attention probability. This immediately tells us that $\{1, 4\}$ cannot form a cluster because there is no such alternative for 1 in the menu $\{b, c\}$, where her choice probability of choosing that alternative is greater than the average choice probability of 1 and 4 of choosing that alternative. By the same argument, it also follows that {1,2,3,4} cannot be part of a cluster. Note that neither $p_1(b,\, bc)$ nor $p_1(c,\, bc)$ would be greater than the averages across the four individuals.

Now look at the possibility of $\{1, 3\}$ forming a cluster. Since $p_1(b,\, bc) > p_3(b,\, bc)$ and $p_3(c,\, bc) > p_1(c,\, bc)$, we can conclude that $b \succ_1 c$ and $c \succ_3 b$ if 1 and 3 are in the same cluster. Since $p_3(b,\, bc) = 2/5$ and $p_1(b,\, bc) = 1/2$, the values for $p_1(c,\, bc)$ and $p_3(c,\, bc)$ must satisfy the following equations. The first of them is

$$\frac{\frac{p_3(b,\,bc)}{1-p_3(c,\,bc)} - \frac{1}{2}\left(p_1(b,\, bc) + p_3(b,\, bc)\right)}{1 - \frac{1}{2}(p_1(b,\, bc) + p_3(b,\, bc))} = \frac{\frac{2}{5(1-p_3(c,\,bc))} - \frac{9}{20}}{\frac{11}{20}} \approx 0.279$$

The second is

$$\frac{p_1(b,\, bc) - \frac{1}{2}\left(p_1(b,\, bc) + p_3(b,\, bc)\right)}{1 - \frac{1}{2}\left(p_1(b,\, bc) + p_3(b,\, bc)\right)} = \frac{\frac{1}{2} - \frac{9}{20}}{\frac{11}{20}} = \frac{1}{11}$$

The above equations come from rearranging the attention probability equations for each of the alternatives to yield an expression for β_i in terms of a menu A and some alternative $x \in A$

$$\beta_i = \frac{\gamma_i(x,\, A) - \mu_{N(i)}(x,\, A)}{1 - \mu_{N(i)}(x,\, A)}$$

However, plugging this back into the equations for $p_1(c,\, bc)$ and $p_3(c,\, bc)$, we find that those probabilities are not consistent with a PIA equilibrium, implying that 1 and 3 cannot be part of the same cluster either. This leaves $N = \{\{1,\, 2\}\,,\, \{3,\, 4\}\}$ as the only possible clustering.

Consider preferences

1: $a \succ_1 c \succ_1 b$

2: $a \succ_2 b \succ_2 c$
3: $a \succ_3 c \succ_3 b$
4: $a \succ_4 b \succ_4 c$
Then note that for values of $\beta_1 = \frac{847-29\sqrt{805}}{477+45\sqrt{805}} \approx 0.014$, $\beta_2 = 1/9$, $\beta_3 = \frac{869-36\sqrt{341}}{495+66\sqrt{341}} \approx 0.119$, $\beta_4 = 1/11$, it can be verified that $p_i(x, A) = \gamma_i(x, A) \prod_{y \succ_i x}(1 - \gamma_i(y, A))$ with $\gamma_i(x, A) = \beta_i + (1 - \beta_i)\mu_{N(i)}(x, A)$. Then, this profile has a PIA representation, and by the uniqueness of the clusters, is unique.

However, note that this representation does not satisfy the homophily condition, as the set of top-agreeable menus for both clusters is $\{\{a, b\}, \{a, c\}, \{a, b, c\}\}$, with the tops being the same. This shows that homophily is not a necessary restriction on the representation for the unique identification of parameters.

Example A.2. Suppose $I = \{1, 2\}$ with both individuals in a single cluster. $X = \{a, b, c\}$ and the analyst has choice data on the menus $\mathcal{X} = \{\{a, b\}, \{a, c\}, \{a, b, c\}\}$, given by $p_i(a, ab) = p_i(a, ac) = p_i(a, X) = 1$. Then, it is straightforward to verify that the preference profile $a \succ_1 b \succ_1 c$, and $a \succ_2 c \succ_2 b$, along with *any* $\beta_i \in (0, 1)$ represent this collection of choice probabilities. While that is enough to establish that the parameters of the model are not uniquely identified, we can go a step further and note that if 1 and 2's preferences were interchanged, we would still obtain parameters that represent the dataset we have.

A.10 Bias of LIM estimator

In this section, we show that under a simplifying assumption that $\beta_i = \beta$ for all $i = 1, ..., n$, the LIM estimator in Section 7 is asymptotically biased. First, observe that when β is common, we can write attention probabilities for each alternative as a function of the number of individuals who prefer e_H to e_L. Let K_t be this random variable for each period (as preferences are randomly determined). The regression we seek to analyze is of the form[36]

$$\mathbb{E}\left[e_{it}|(\succ_{jt})\right] = \alpha + \lambda \bar{E}_t + u_{it}$$

where the expectation on the left hand side is conditional on the preference profile in period t. Particularly, let $\gamma_H(K)$ and $\gamma_L(K)$ be attention probabilities for e_H and e_L given K. Define $h_H(K) = e_H\gamma_H(K) + e_L\gamma_L(K)(1 - \gamma_H(K))$ and $h_L(K) = e_H\gamma_H(K)(1 - \gamma_L(K)) + e_L\gamma_L(K)$. Then $\mathbb{E}\left[e_{it}|(\succ_{jt})\right] = h_H(K_t)$ if $e_H \succ_{it} e_L$ and $h_L(K_t)$ otherwise. Let $g(K_t) = \frac{K_t}{n}h_H(K_t) + \frac{n-K_t}{n}h_L(K_t) = \frac{1}{n}\sum_{i=1}^{n}\mathbb{E}\left[e_{it}\right]$.

Note that $\mathbb{E}\left[e_{it}|K_t\right] = g(K_t)$. By the identical randomization of preferences, given only the number of individuals with $e_H \succ_{it} e_L$, all individuals' conditional expected

[36]We run the regression for each i individually, so notions of asymptotics are for when $T \to \infty$

efforts are in fact the average expected effort. Further, since conditioning on $(\succ_{jt})$ also conditions on K_t, by the law of iterated expectations, $\mathbb{E}\left[e_{it}|K_t\right] = \mathbb{E}\left[\mathbb{E}\left[e_{it}|\left(\succ_{jt}\right)\right]|K_t\right]$, implying $\mathbb{E}\left[e_{it}\right] = \mathbb{E}\left[g(K_t)\right]$. Then,

$$\begin{aligned}
\mathrm{Cov}(g(K_t), \mathbb{E}\left[e_{it}|\left(\succ_{jt}\right)\right]) &= \mathbb{E}\left[\mathbb{E}\left[g(K_t)\,\mathbb{E}\left[e_{it}|\left(\succ_{jt}\right)\right]|K_t\right]\right] - \mathbb{E}\left[g(K_t)\right]\mathbb{E}\left[\mathbb{E}\left[e_{it}|\left(\succ_{jt}\right)\right]\right] \\
&= \mathbb{E}\left[g(K_t)^2\right] - \mathbb{E}\left[g(K_t)\right]^2 \\
&= \mathrm{Var}(g(K_t))
\end{aligned}$$

Therefore, when $\bar{E}_t = g(K_t)$, $\lambda = 1$.

To avoid this mechanical regression coefficient in the case where one's own expected effort is included in the cluster average, consider the case where we leave the individual out of the average. By rearranging (3), we get that the coefficient of interest when regressing $\mathbb{E}\left[e_{it}|(\succ_{jt})\right]$ on $\frac{1}{n-1}\sum_{i'\neq i}\mathbb{E}\left[e_{i't}|\left(\succ_{jt}\right)\right] = \frac{n}{n-1}g(K_t) - \frac{1}{n-1}\mathbb{E}\left[e_{it}|\left(\succ_{jt}\right)\right]$ is $\frac{(1-\beta)(n-1)}{n-1+\beta}$. Let $W = \mathbb{E}\left[\mathrm{Var}(\mathbb{E}\left[e_{it}|(\succ_{jt})\right]|K_t)\right]$ and $V = \mathrm{Var}(g(K_t))$. By the law of total variance, $\mathrm{Var}(\mathbb{E}\left[e_{it}|(\succ_{jt})\right]) = V + W$. Then,

$$\begin{aligned}
\mathrm{Cov}(\mathbb{E}\left[e_{it}|\left(\succ_{jt}\right)\right], \bar{E}_t) &= \frac{n}{n-1}\mathrm{Cov}(\mathbb{E}\left[e_{it}|\left(\succ_{jt}\right)\right], g(K_t)) - \frac{1}{n-1}\mathrm{Var}(\mathbb{E}\left[e_{it}|\left(\succ_{jt}\right)\right]) \\
&= V - \frac{1}{n-1}W \\
\mathrm{Var}(\bar{E}_t) &= \frac{n^2}{(n-1)^2}V + \frac{1}{(n-1)^2}(V+W) - 2\frac{n}{(n-1)^2}V \\
&= V + \frac{1}{(n-1)^2}W
\end{aligned}$$

Given that $\lambda = \frac{(1-\beta)(n-1)}{n-1+\beta}$, i.e. $1-\beta = \frac{\lambda n}{n-1+\lambda}$, if the LIM coefficient on $\bar{E}_t$ is a reasonable estimator for λ, then the same transformation on the LIM estimator is a reasonable estimator of $1-\beta$. Furthermore, this $1-\beta$ is increasing in λ. Therefore, if LIM overestimates λ, so does the transformation overestimate $1-\beta$. Then, observe that the population LIM coefficient is greater than λ if and only if

$$\begin{aligned}
\frac{(n-1)V - W}{(n-1)^2V + W} &> \frac{1-\beta}{n-1+\beta} \\
\iff \frac{V}{W} &> \frac{1}{(n-1)\beta}
\end{aligned}$$

A.11 Social multiplier

We start with the attention probabilities of each alternative as a function of the number of individuals with $e_H \succ_i e_L$ as the solution to the system

$$\gamma_H(k) = \beta + (1-\beta)\frac{k\gamma_H(k) + (n-k)\gamma_H(k)(1-\gamma_L(k))}{n}$$
$$\gamma_L(k) = \beta + (1-\beta)\frac{k\gamma_L(k)(1-\gamma_H(k)) + (n-k)\gamma_L(k)}{n}$$

Define a function $F : [0,\, 1]^2 \to [0,\, 1]^2$ such that

$$F_1(\gamma_H,\, \gamma_L) = \beta + (1-\beta)\frac{k\gamma_H + (n-k)\gamma_H(1-\gamma_L) + 1}{n+1}$$
$$F_2(\gamma_H,\, \gamma_L) = \beta + (1-\beta)\frac{k\gamma_L(1-\gamma_H) + (n-k)\gamma_L}{n+1}$$

Then, starting with γ_H, γ_L under the original PIA equilibrium, $(\gamma_H^d,\, \gamma_L^d) = F(\gamma_H,\, \gamma_L)$ is the direct effect on attention probabilities. Further, the new PIA equilibrium is the fixed point of the function F. Observe that

$$F_1(\gamma_H(k),\, \gamma_L(k)) = \frac{n}{n+1}\gamma_H(k) + \frac{1}{n+1} \geq \gamma_H(k)$$
$$F_2(\gamma_H(k),\, \gamma_L(k)) = \frac{n}{n+1}\gamma_L(k) \leq \gamma_L(k)$$

with the inequalities turning strict for all $k \neq n$. Then, observe that F_1 is increasing in γ_H and decreasing in γ_L. F_2 is increasing in γ_L and decreasing in γ_H. Let $\{\gamma_H^t,\, \gamma_L^t\}_{t=0}^{\infty}$ be defined as $(\gamma_H^0,\, \gamma_L^0) = (\gamma_H(k),\, \gamma_L(k))$ and $(\gamma_H^{t+1},\, \gamma_L^{t+1}) = F(\gamma_H^t,\, \gamma_L^t)$. We know $\gamma_H^1 > \gamma_H^0$ and $\gamma_L^1 < \gamma_L^0$, so by induction, γ_H^t is an increasing sequence, while γ_L^t is a decreasing sequence. Since each individual sequence is monotone and bounded, they converge. Since F is continuous, they converge to a fixed point of the function, i.e. the new PIA equilibrium. The direct effect on any outcomes of interest is calculated w.r.t. γ_H^1 and γ_L^1. The total effect is calculated w.r.t to the limiting attention probabilities, the new PIA equilibrium.

Choice probabilities. One outcome of interest is μ_H, the average probability of choosing e_H. We calculate the effect only on the individuals originally in the cluster, i.e. the initial n individuals. We can construct a sequence μ_H^t, where

$$\mu_H^t = \frac{k\gamma_H^t + (n-k)\gamma_H^t(1-\gamma_L^t)}{n}$$

By the properties of the attention probability sequences, μ_H^t is an increasing sequence. Then, both the direct and total effect on μ_H are positive, with the total effect being larger than the direct effect. Hence, the PIA model admits a social multiplier with respect to

μ_H. By a similar argument, the social multiplier exists for μ_L as well.

Average effort. The average effort in the cluster is defined as

$$\begin{aligned}\mathbb{E}_\mu[e] &= e_H\mu_H + e_L\mu_L \\ &= e_H\gamma_H(k/n + (n-k)(1-\gamma_L)/n) + e_L\gamma_L(k(1-\gamma_H)/n + (n-k)/n) \\ &= e_H\gamma_H + e_L\gamma_L - \gamma_L\gamma_H\left(\frac{(n-k)e_H + ke_L}{n}\right)\end{aligned}$$

Observe that

$$\frac{\partial\,\mathbb{E}_\mu[e]}{\partial\gamma_H} = e_H - \gamma_L\left(\frac{(n-k)e_H + ke_L}{n}\right) > 0$$

However,

$$\frac{\partial\,\mathbb{E}_\mu[e]}{\partial\gamma_L} = e_L - \gamma_H\left(\frac{(n-k)e_H + ke_L}{n}\right)$$

The sign of this derivative is ambiguous, and depends on initial conditions. Let the direct effect on choice probabilities be $\Delta^1\mu_H = \mu_H^1 - \mu_H^0$ and $\Delta^1\mu_L = \mu_L^1 - \mu_L^0$. Using the formula for average choice probabilities as a function of attention probabilities, we get

$$\begin{aligned}\Delta^1\mu_H &= \frac{n-k}{(n+1)^2}\gamma_H^0\gamma_L^0 + \frac{k + (n-k)\left(1 - \frac{n}{n+1}\gamma_L^0\right)}{n(n+1)} - \frac{1}{n+1}\mu_H^0 \\ \Delta^1\mu_L &= -\left[\frac{1}{n+1}\mu_L^0 + \frac{k}{(n+1)^2}(1-\gamma_H^0)\gamma_L^0\right]\end{aligned}$$

When $k = 0$, observe that $\Delta^1\mu_H = \frac{n\beta+1}{(n+1)^2}$ and $\Delta^1\mu_L = -\frac{1}{n+1}$. Then, for small $e_H - e_L$, the direct effect of the intervention is negative on average expected effort. We can, however, provide a sufficient condition for when both the direct and indirect effects are positive, and the social multiplier exists. We have already noted that the sequence of attention probabilities are monotone. Then, it is enough that $\frac{\partial\,\mathbb{E}_\mu[e]}{\partial\gamma_L} \leq 0$ for the direct effect to be positive and the total effect to be larger. Using $\gamma_H \geq \beta$,

$$\begin{aligned}\frac{\partial\,\mathbb{E}_\mu[e]}{\partial\gamma_L} &\leq e_L - \beta\left(\frac{(n-k)e_H + ke_L}{n}\right) \\ 0 &\geq e_L - \beta\left(\frac{(n-k)e_H + ke_L}{n}\right) \\ \iff \beta &\geq \frac{e_L}{\frac{k}{n}e_L + \frac{n-k}{n}e_H}\end{aligned} \tag{SM}$$

Then, if (SM) holds, the direct effect and indirect effect are both positive, and the social multiplier exists.

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